Matthias Krause
Alexander Moch

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Exercise Sheet 5

Exercise 5.1:
Show that $\leq_p$ is a reflexive and transitive relation over $\mathcal{NP}$.

Hint: W.l.o.g., you can assume here that the polynomials which bound the worst-case running time of the corresponding transformations are monotonically increasing.

Exercise 5.2:
Show that PARTITION $\in \mathcal{NPC}$ by showing that KP* $\leq_p$ PARTITION.
(KP* $\in \mathcal{NPC}$ has already been shown in detail in the lecture.)

Exercise 5.3:
Show: The optimization versions of TSP and KP are $\mathcal{NP}$-easy.

Exercise 5.4:
For the BIN PACKING PROBLEM (BPP), each instance consists of a set of objects $A = \{a_1, \ldots, a_n\}$, a function $s : A \to \mathbb{N}$, which assigns a size/weight to each of the objects, and a container size $B \in \mathbb{N}$. The question asked in the optimization version is, how these objects can be packed into as few containers of capacity B as possible; more precisely, we look for a partition $A_1 \,\dot{\cup}\, \cdots \,\dot{\cup}\, A_m = A$ such that $\max\{\sum_{a_i \in A_j} s(a_i) \mid 1 \leq j \leq m\} \leq B$ holds and m is as small as possible. In the decision version, we are given an additional number $K \in \{1, \ldots, n\}$ and ask whether K containers are sufficient for storing all objects. Show:

a) The decision version of BPP is $\mathcal{NP}$-complete.

b) The optimization version of BPP is $\mathcal{NP}$-equivalent.

Exercise 5.5:
Which of the following statements are true? Why?

a) $L_1 \leq_p L_2 \Rightarrow \overline{L}_1 \leq_p \overline{L}_2$,

b) $(L_1 \leq_p L_2 \;\wedge\; L_2 \in \mathcal{P}) \Rightarrow L_1 \in \mathcal{P}$,

c) $(L_1 \leq_p L_2 \;\wedge\; L_2 \in \mathcal{NP}) \Rightarrow L_1 \in \mathcal{NP}$,

d) $(L \in \mathcal{NP} \;\wedge\; L \notin \mathcal{P}) \Rightarrow \mathcal{P} \cap \mathcal{NPC} = \emptyset$.