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Exercise Sheet 4

Exercise 4.1:
Given two languages $L_1 \subseteq \Sigma_1^*$ and $L_2 \subseteq \Sigma_2^*$ with $L_1 \in \mathcal{P}$ and $\emptyset \neq L_2 \neq \Sigma_2^*$, show: $L_1 \leq_p L_2$. Would $\mathcal{P} = \mathcal{NP}$ imply that all languages in $\mathcal{P}$ are $\mathcal{NP}$-complete?

Exercise 4.2:
Show that $\mathcal{NP}$ is closed w.r.t. $\leq_p$, i.e., from $L_1 \leq_p L_2$ and $L_2 \in \mathcal{NP}$ follows $L_1 \in \mathcal{NP}$.

Exercise 4.3:
We call two languages $L_1 \subseteq \Sigma_1^*$ and $L_2 \subseteq \Sigma_2^*$ polynomial-time isomorphic if there is a bijective function $f : \Sigma_1^* \to \Sigma_2^*$ such that f is a polynomial reduction from L_1 to L_2 and f^{-1} is a polynomial reduction from L_2 to L_1.
Show that the following decision problems (treated as languages) are polynomial-time isomorphic. The respective instances consist of an undirected graph $G = (V, E)$ and a number $K \in \{0, \ldots, |V|\}$.

Independent Set: G contains an independent set (i.e., a set of vertices such that for every two vertices in the set, there is no edge connecting the two) of size at least K.

Clique: G contains a clique of size at least K.

Vertex Cover: G contains a vertex cover (i.e., a set of vertices such that each edge of the graph is incident to / "contains" at least one vertex of the set) of size at most K.

Exercise 4.4:
In this exercise, the inputs are DNF-formulas (i.e., disjunctions $M_1 \vee \cdots \vee M_n$ of conjunctions $L_1 \wedge \cdots \wedge L_s$ of literals L_i). Assume $\mathcal{P} \neq \mathcal{NP}$ and decide for each of the following two problems whether it is in $\mathcal{P}$:

a) Is there an assignment to the variables of the DNF-formula given as an input such that it evaluates to $\mathbf{1}$?

b) Is there an assignment to the variables of the DNF-formula given as an input such that it evaluates to $\mathbf{0}$?

Exercise 4.5:
The Directed Hamiltonian Circuit Problem (DHC) for directed graphs is defined analogously to the Hamiltonian Circuit Problem (HC) for undirected graphs, which has already been introduced in the lecture.
Prove that HC is $\mathcal{NP}$-complete by showing that $DHC \leq_p HC$. $HC \in \mathcal{NP}$ and $DHC \in \mathcal{NPC}$ can be assumed as known facts in this exercise.