Algorithmics

Spring ’20 - Tutorial #3

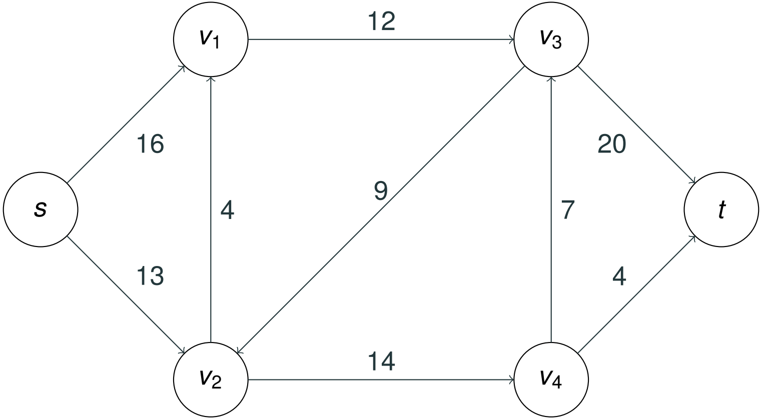
Alexander Moch

April 30, 2020

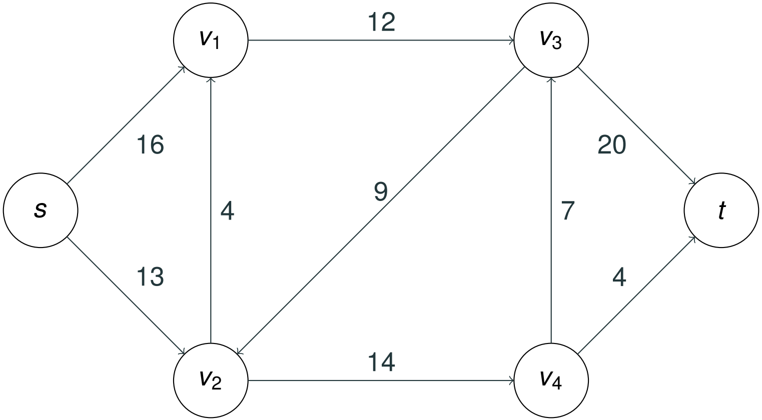
Exercise 3.1

Task

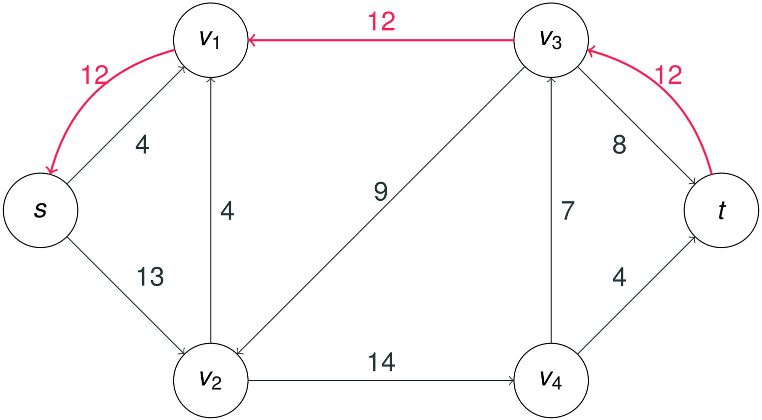
Show the execution of the Edmonds-Karp algorithm on the following flow network with source s and sink t:



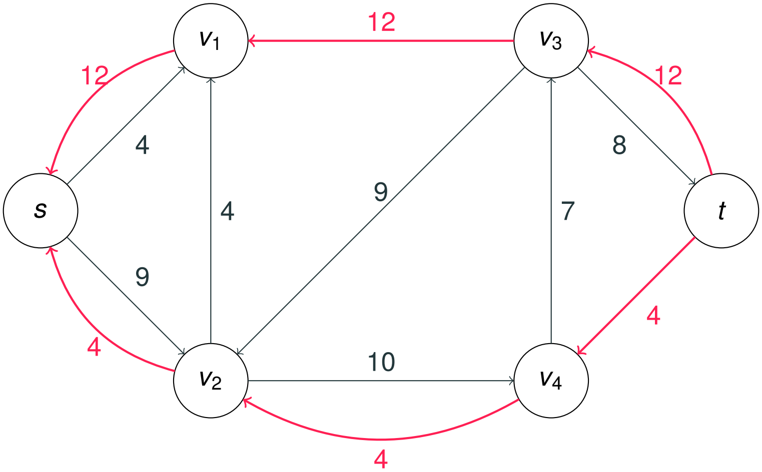
Solution



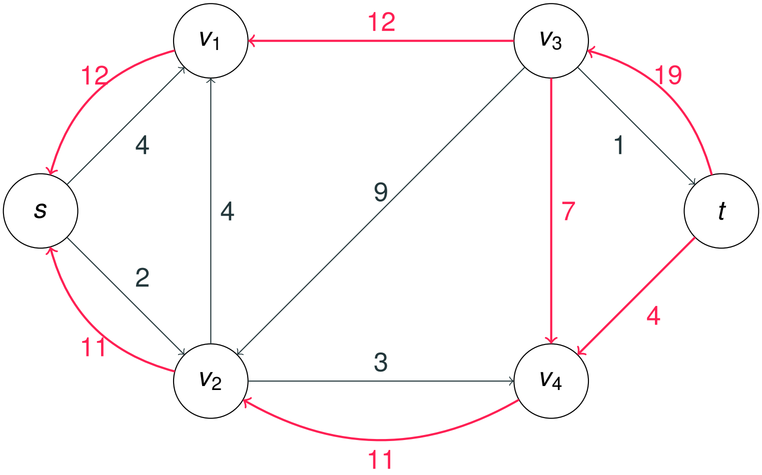
Solution



Solution

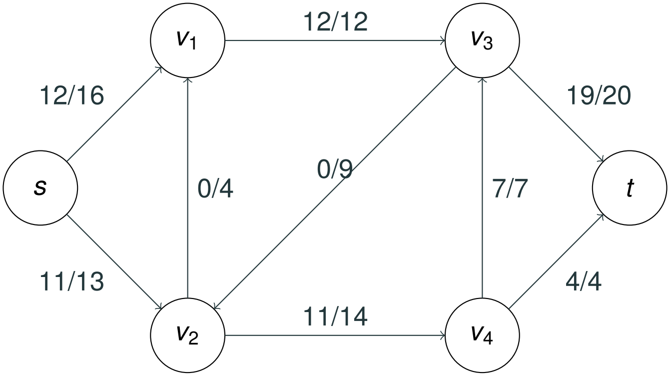


Solution



Solution

Maximum Flow:

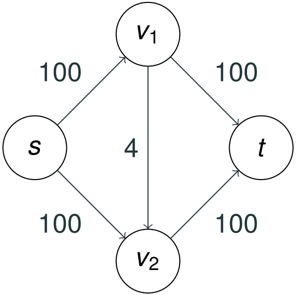


Value/size of the maximum flow: 23

Exercise 3.2

Task

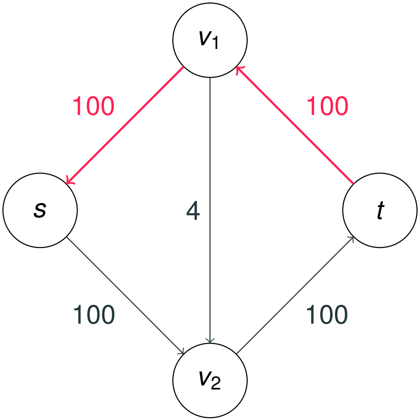
Consider the following flow network with source s and sink t:



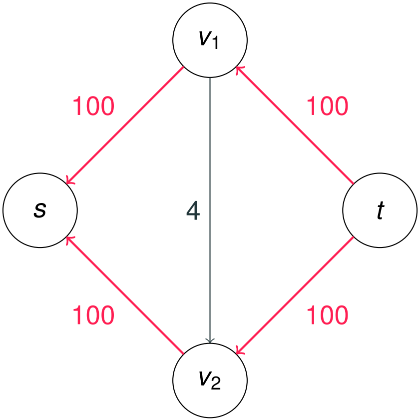
a) Show the execution of the Edmonds-Karp algorithm on this flow network.

b) What is the worst-case number of possible iterations for the (general) Ford-Fulkerson method in this case?

Solution (a)

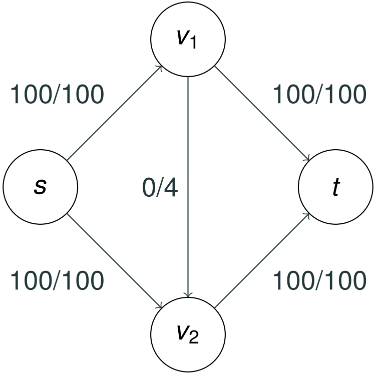


Solution (a)



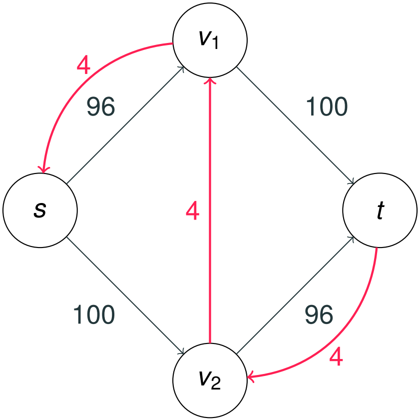
Solution (a)

Maximum Flow:

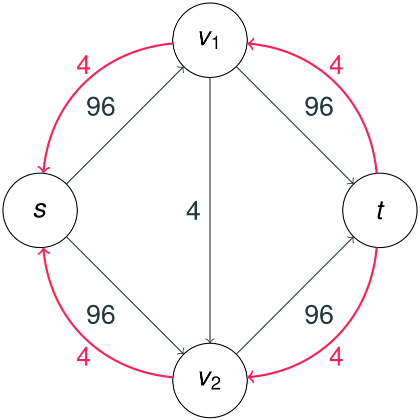


Value/size of the maximum flow: 200

Solution (b)



Solution (b)



Solution (b)

...

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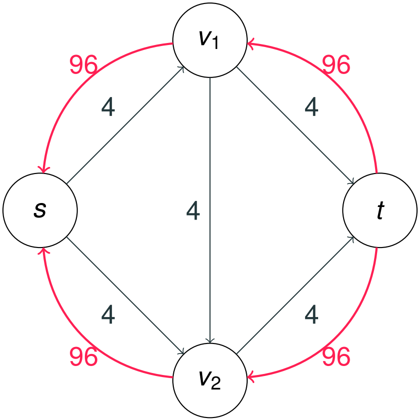
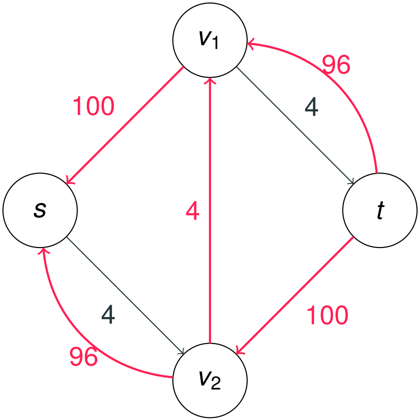
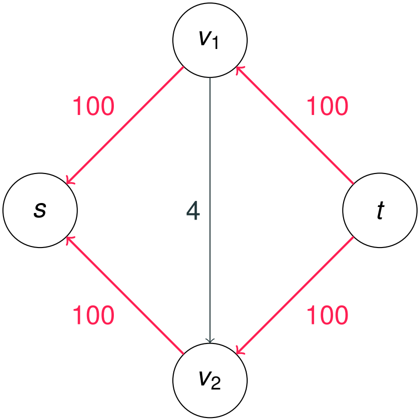


Solution (b)

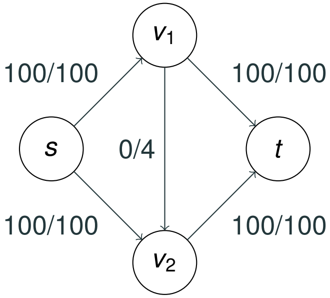


Solution (b)



Solution (b)

Maximum Flow:



Value/size of the maximum flow: 200

Worst-case improvement per iteration: 4

$\Rightarrow$ **Worst-case number of iterations:** $\frac{200}{4} = 50$

Exercise 3.3

Task

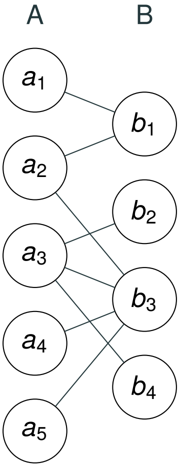
Find a maximum bipartite matching for the undirected Graph $G = (V, E)$, defined by the vertex partition $V = A \cup B$, $A = \{a_1, \ldots, a_5\}$, $B = \{b_1, \ldots, b_4\}$ and

$$\begin{aligned} E =& \{\{a_1, b_1\}, \{a_2, b_1\}, \{a_2, b_3\}, \{a_3, b_2\}\{a_3, b_3\}, \{a_3, b_4\}, \\ & \{a_4, b_3\}, \{a_5, b_3\}\}. \end{aligned}$$

In each iteration, pick the augmenting path that is lexicographically smallest.

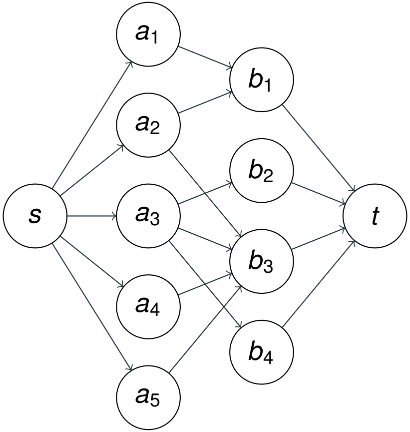
Solution

Bipartite input graph:



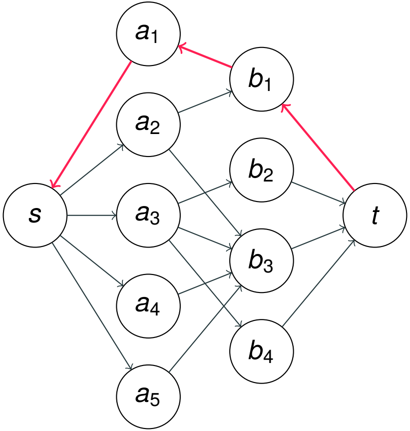
Solution

Corresponding flow network:

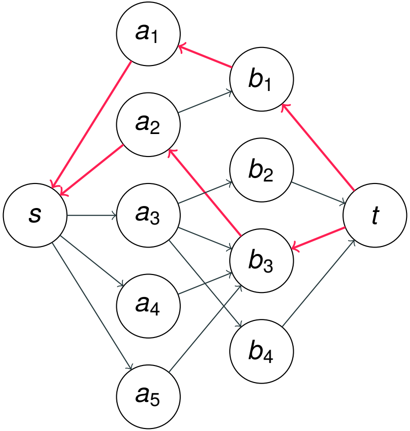


Note: Each edge has weight 1*.*

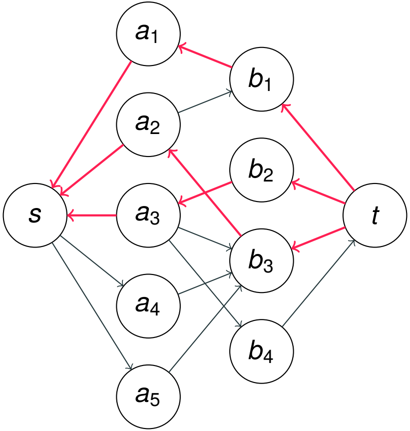
Solution



Solution

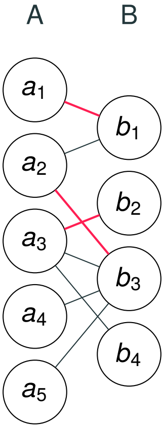


Solution



Solution

As there is no further augmenting path, we obtain the maximum matching $M = \{\{a_1, b_1\}, \{a_2, b_3\}, \{a_3, b_2\}\} \subseteq E$ for G.



Exercise 3.4

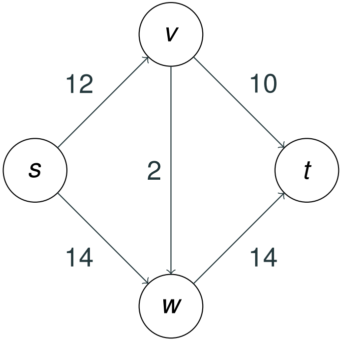
Task

For each of the following flow networks with source s and sink t provide a minimal cut (S, T) and its capacity $c\,(S, T)$.

a) See exercise sheet or corresponding solution slide.

b) See exercise sheet or corresponding solution slide.

Solution (a)

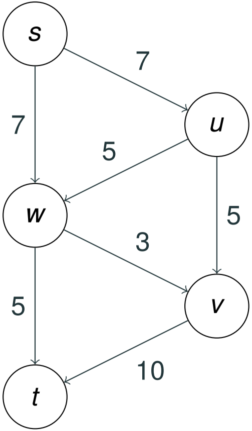


S	T	c(S,T)
$\{s\}$	$\{v, w, t\}$	26
$\{s, v\}$	$\{w, t\}$	26
$\{s, w\}$	$\{v, t\}$	26
$\{s, v, w\}$	$\{t\}$	24

Minimal cut: $S = \{s, v, w\}$, $T = \{t\}$

Capacity: $c(S, T) = 24$

Solution (b)



S	T	c(S,T)
$\{s\}$	$\{u, v, w, t\}$	14
$\{s, u\}$	$\{v, w, t\}$	17
$\{s, v\}$	$\{v, w, t\}$	24
$\{s, w\}$	$\{u, v, t\}$	15
$\{s, u, v\}$	$\{w, t\}$	22
$\{s, u, w\}$	$\{v, t\}$	13
$\{s, v, w\}$	$\{u, t\}$	22
$\{s, u, v, w\}$	$\{t\}$	15

Minimal cut: $S = \{s, u, w\}$, $T = \{v, t\}$

Capacity: $c(S, T) = 13$