

# Algorithmics

Spring '20 - Tutorial #3

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Alexander Moch

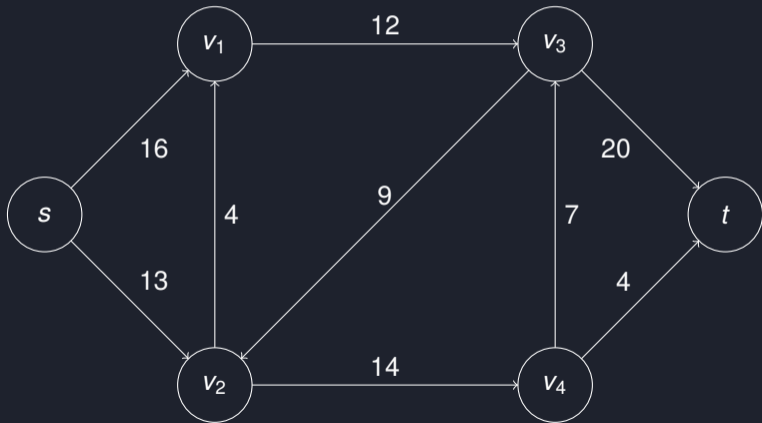
April 30, 2020

## Exercise 3.1

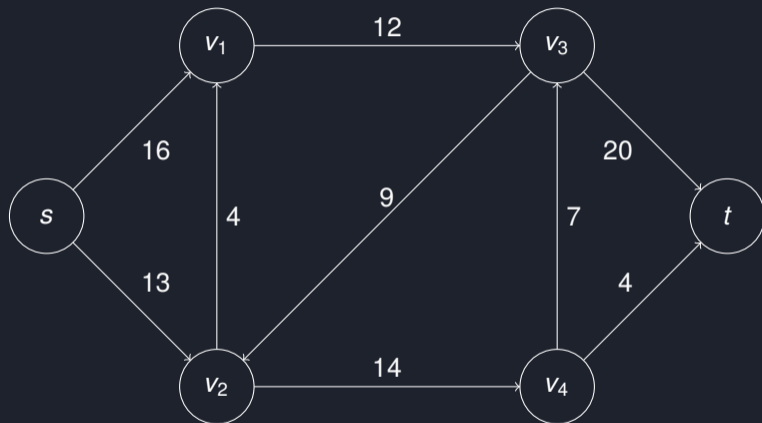
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## Task

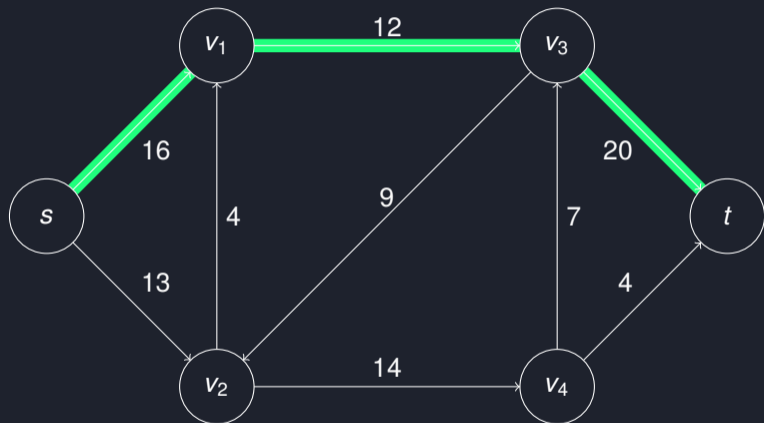
Show the execution of the Edmonds-Karp algorithm on the following flow network with source  $s$  and sink  $t$ :



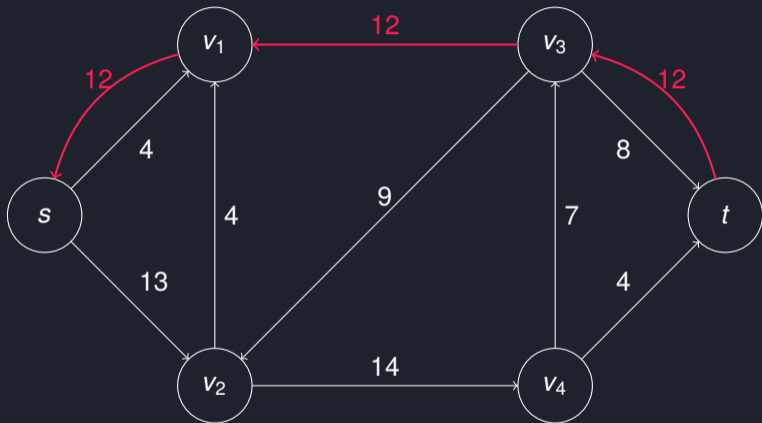
# Solution



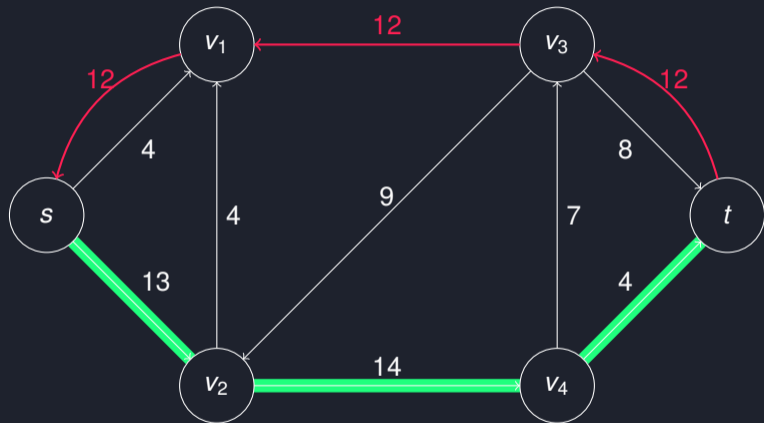
# Solution



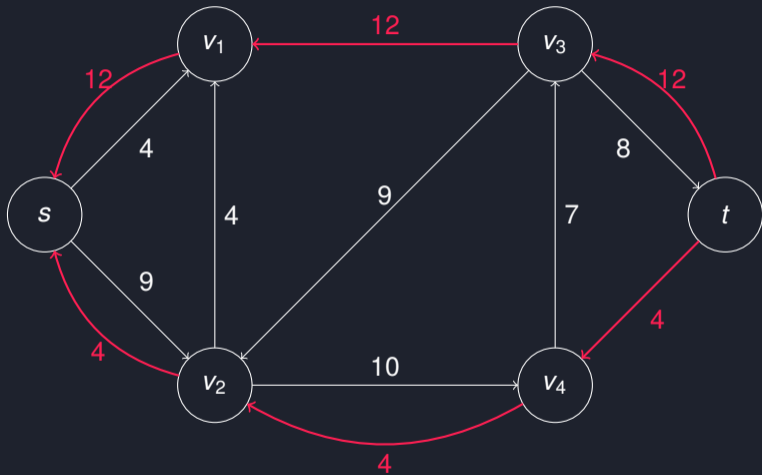
# Solution



# Solution

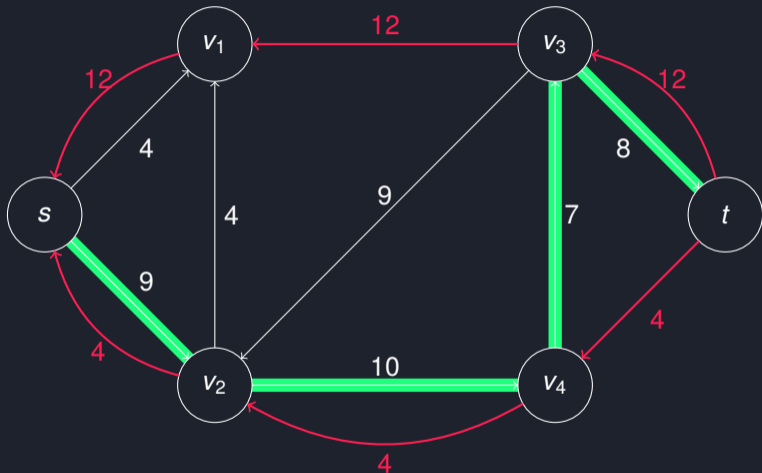


# Solution

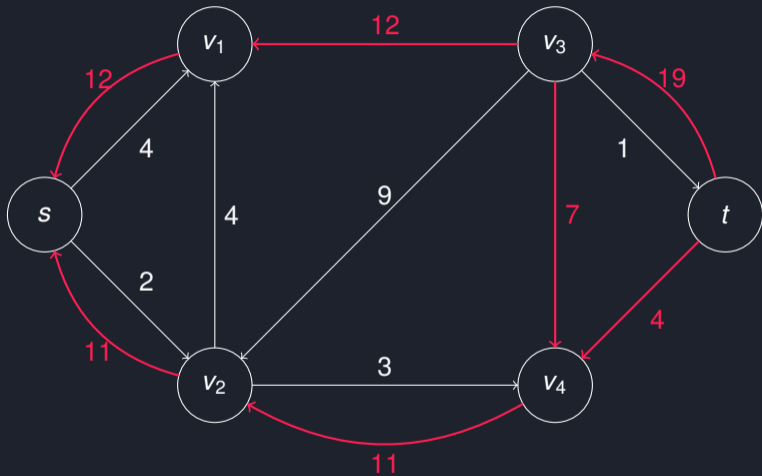




# Solution

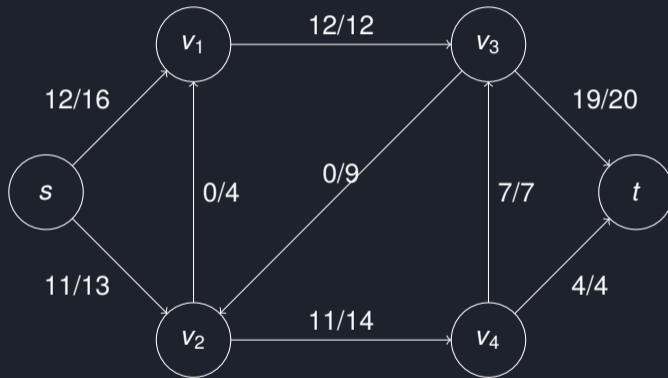


# Solution



# Solution

Maximum Flow:



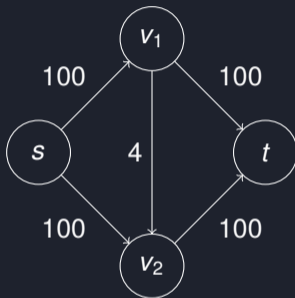
Value/size of the maximum flow: 23

## Exercise 3.2

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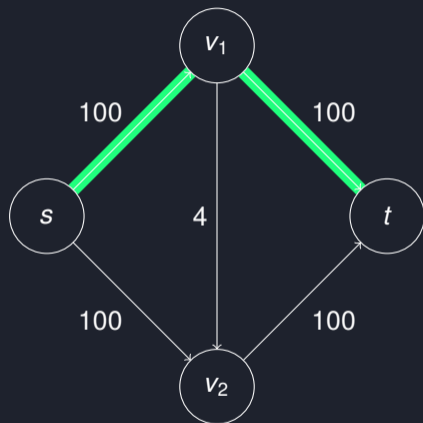
## Task

Consider the following flow network with source  $s$  and sink  $t$ :

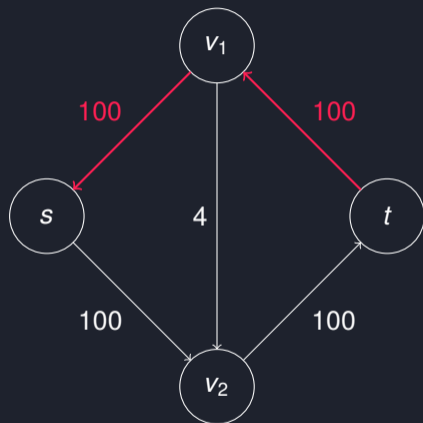


- Show the execution of the Edmonds-Karp algorithm on this flow network.
- What is the worst-case number of possible iterations for the (general) Ford-Fulkerson method in this case?

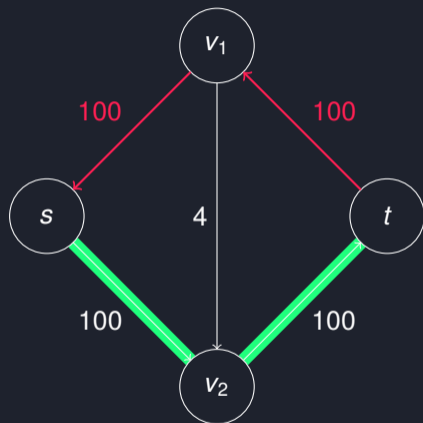
## Solution (a)



## Solution (a)

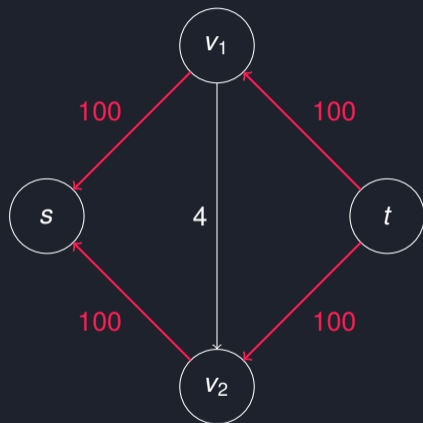


## Solution (a)



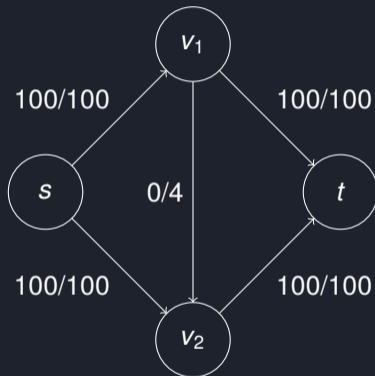


## Solution (a)



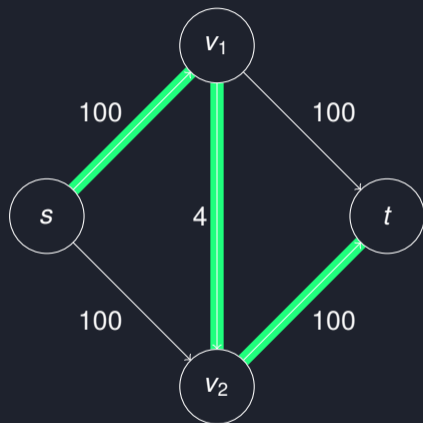
## Solution (a)

Maximum Flow:

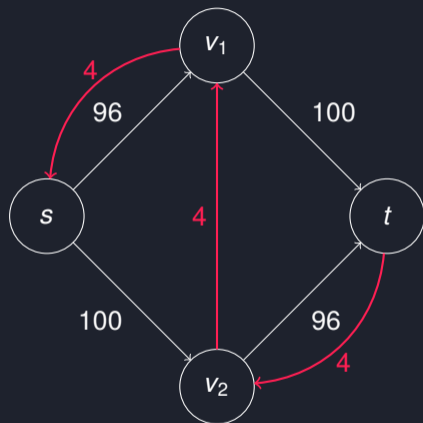


Value/size of the maximum flow: 200

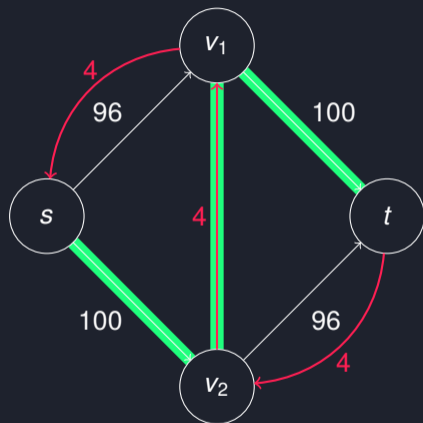
## Solution (b)



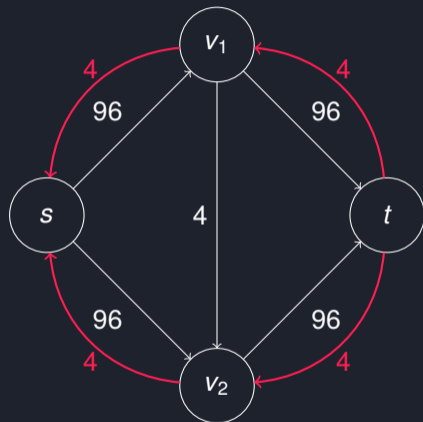
## Solution (b)



## Solution (b)



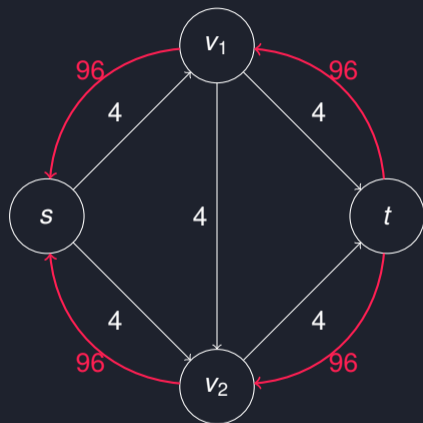
## Solution (b)



## Solution (b)

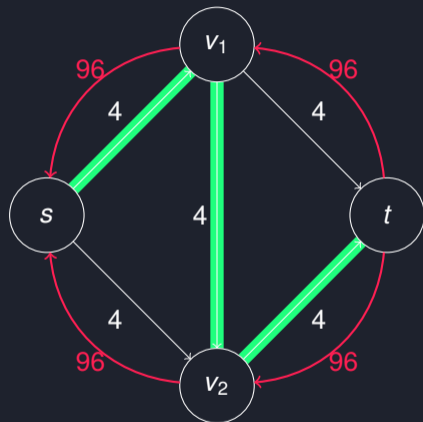
...

## Solution (b)

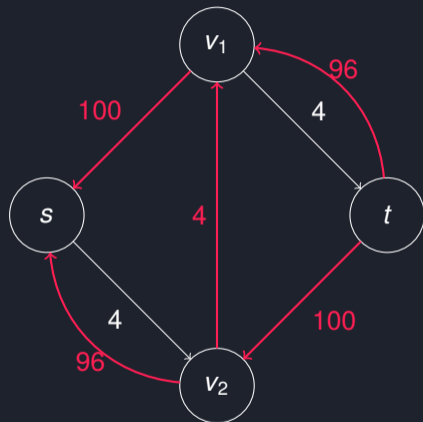




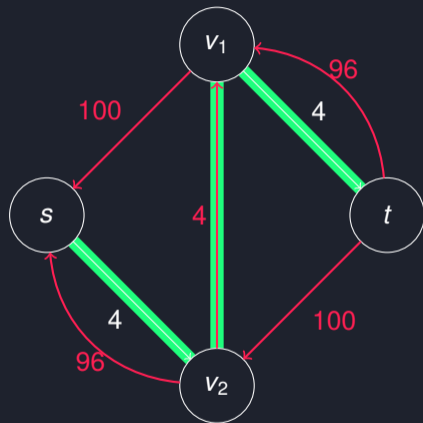
## Solution (b)



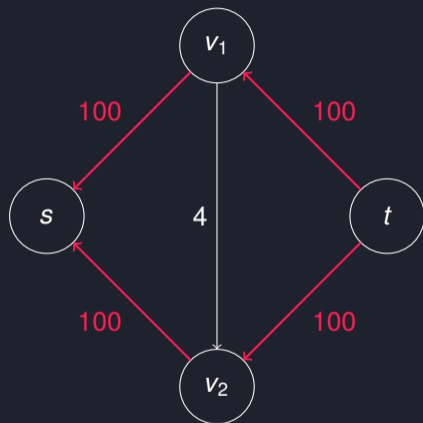
## Solution (b)



## Solution (b)

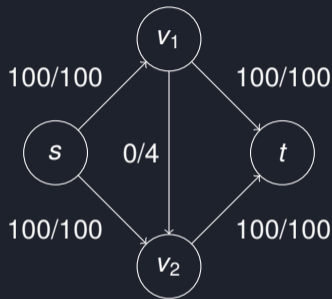


## Solution (b)



## Solution (b)

Maximum Flow:



Value/size of the maximum flow: 200

Worst-case improvement per iteration: 4

$\Rightarrow$  **Worst-case number of iterations:**  $\frac{200}{4} = 50$

## Exercise 3.3

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## Task

Find a maximum bipartite matching for the undirected Graph  $G = (V, E)$ , defined by the vertex partition  $V = A \cup B$ ,  $A = \{a_1, \dots, a_5\}$ ,  $B = \{b_1, \dots, b_4\}$  and

$$E = \{\{a_1, b_1\}, \{a_2, b_1\}, \{a_2, b_3\}, \{a_3, b_2\}, \{a_3, b_3\}, \{a_3, b_4\}, \\ \{a_4, b_3\}, \{a_5, b_3\}\}.$$

In each iteration, pick the augmenting path that is lexicographically smallest.

# Solution

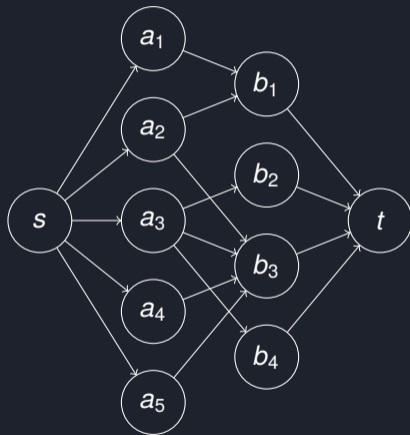
Bipartite input graph:





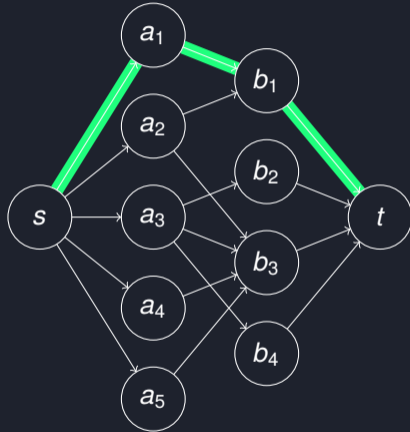
# Solution

Corresponding flow network:

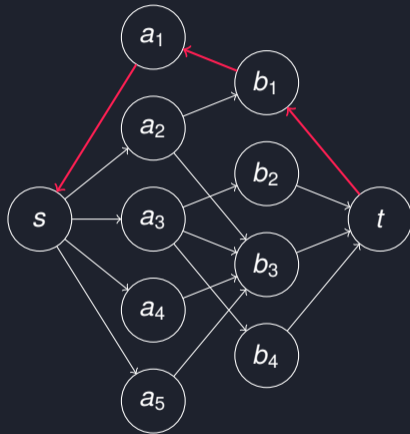


Note: Each edge has weight 1.

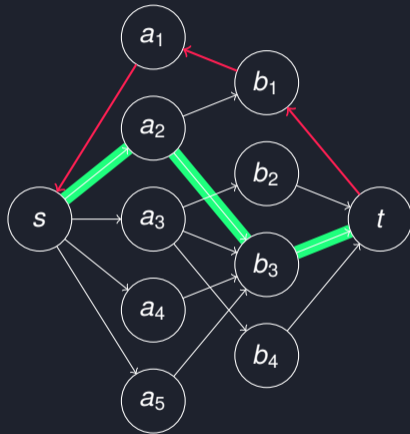
# Solution



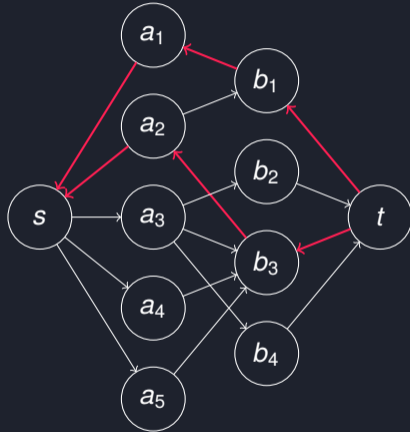
# Solution



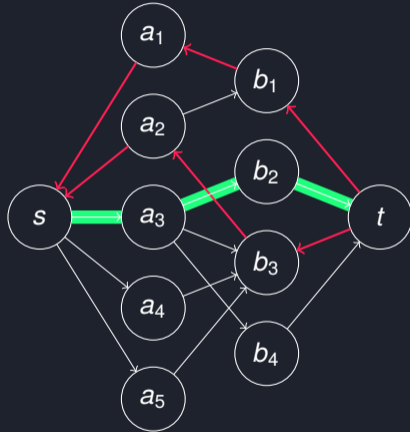
# Solution



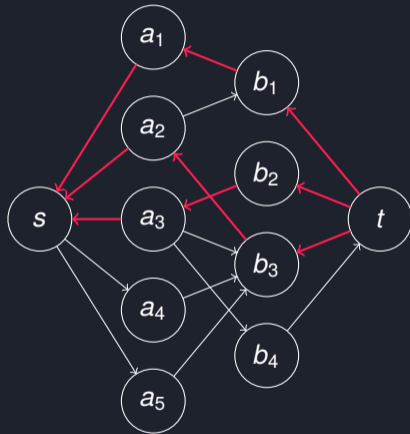
# Solution



# Solution



# Solution



## Solution

As there is no further augmenting path, we obtain the maximum matching  $M = \{\{a_1, b_1\}, \{a_2, b_3\}, \{a_3, b_2\}\} \subseteq E$  for  $G$ .





## Exercise 3.4

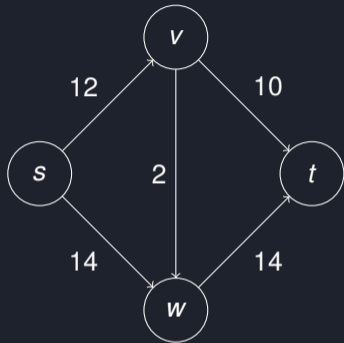
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# Task

For each of the following flow networks with source  $s$  and sink  $t$  provide a minimal cut  $(S, T)$  and its capacity  $c(S, T)$ .

- a) See exercise sheet or corresponding solution slide.
- b) See exercise sheet or corresponding solution slide.

## Solution (a)

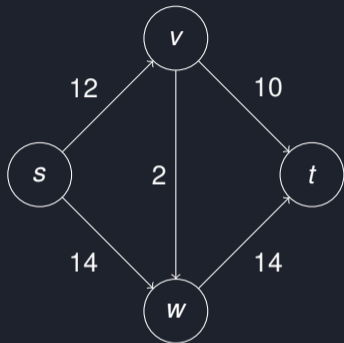


S

T

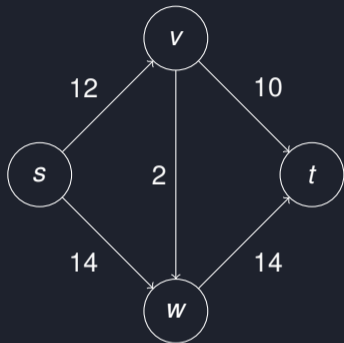
$c(S,T)$

## Solution (a)



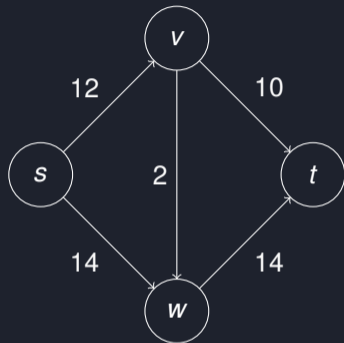
S	T	$c(S,T)$
{s}	{v, w, t}	26

## Solution (a)



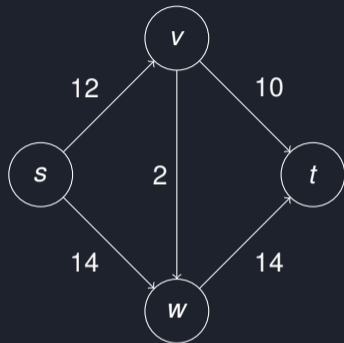
S	T	$c(S,T)$
{s}	{v, w, t}	26
{s, v}	{w, t}	26

## Solution (a)



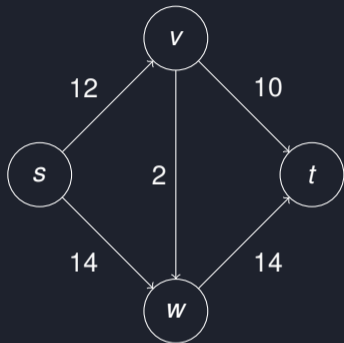
S	T	c(S,T)
{s}	{v, w, t}	26
{s, v}	{w, t}	26
{s, w}	{v, t}	26

## Solution (a)



S	T	c(S,T)
{s}	{v, w, t}	26
{s, v}	{w, t}	26
{s, w}	{v, t}	26
{s, v, w}	{t}	24

## Solution (a)



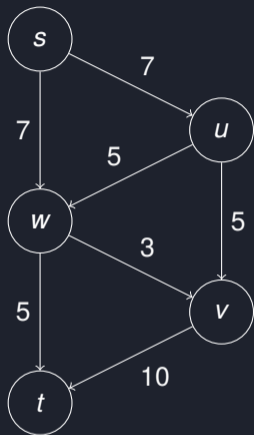
S	T	$c(S,T)$
{s}	{v, w, t}	26
{s, v}	{w, t}	26
{s, w}	{v, t}	26
{s, v, w}	{t}	24

**Minimal cut:**  $S = \{s, v, w\}$ ,  $T = \{t\}$

**Capacity:**  $c(S, T) = 24$

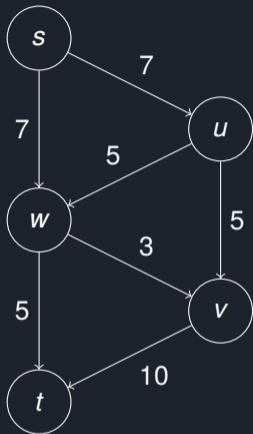


## Solution (b)



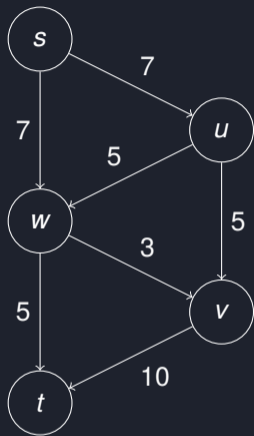
S	T	c(S,T)
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## Solution (b)



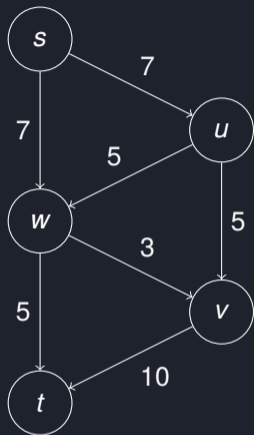
S	T	$c(S,T)$
{s}	{u, v, w, t}	14

## Solution (b)



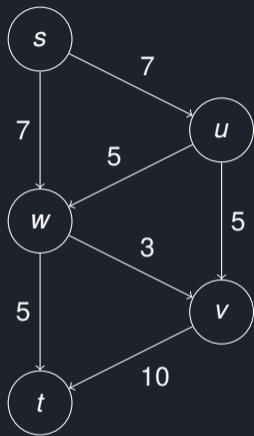
S	T	$c(S,T)$
$\{s\}$	$\{u, v, w, t\}$	14
$\{s, u\}$	$\{v, w, t\}$	17

## Solution (b)



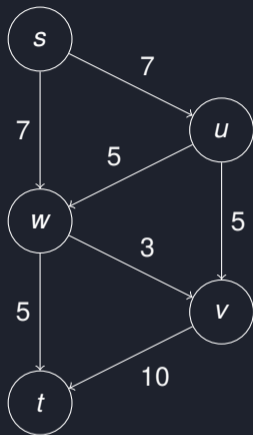
S	T	$c(S,T)$
$\{s\}$	$\{u, v, w, t\}$	14
$\{s, u\}$	$\{v, w, t\}$	17
$\{s, v\}$	$\{v, w, t\}$	24

## Solution (b)



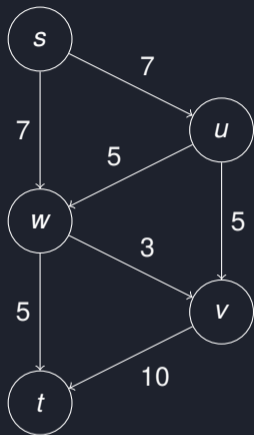
S	T	$c(S,T)$
$\{s\}$	$\{u, v, w, t\}$	14
$\{s, u\}$	$\{v, w, t\}$	17
$\{s, v\}$	$\{v, w, t\}$	24
$\{s, w\}$	$\{u, v, t\}$	15

## Solution (b)



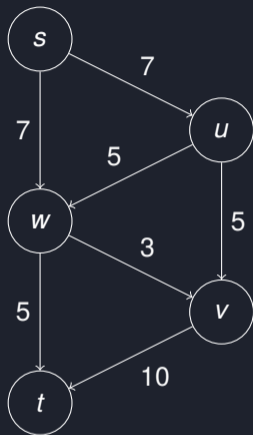
S	T	$c(S,T)$
$\{s\}$	$\{u, v, w, t\}$	14
$\{s, u\}$	$\{v, w, t\}$	17
$\{s, v\}$	$\{v, w, t\}$	24
$\{s, w\}$	$\{u, v, t\}$	15
$\{s, u, v\}$	$\{w, t\}$	22

## Solution (b)



S	T	$c(S,T)$
$\{s\}$	$\{u, v, w, t\}$	14
$\{s, u\}$	$\{v, w, t\}$	17
$\{s, v\}$	$\{v, w, t\}$	24
$\{s, w\}$	$\{u, v, t\}$	15
$\{s, u, v\}$	$\{w, t\}$	22
$\{s, u, w\}$	$\{v, t\}$	13

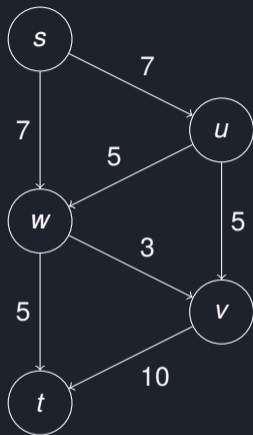
## Solution (b)



S	T	$c(S,T)$
$\{s\}$	$\{u, v, w, t\}$	14
$\{s, u\}$	$\{v, w, t\}$	17
$\{s, v\}$	$\{v, w, t\}$	24
$\{s, w\}$	$\{u, v, t\}$	15
$\{s, u, v\}$	$\{w, t\}$	22
$\{s, u, w\}$	$\{v, t\}$	13
$\{s, v, w\}$	$\{u, t\}$	22

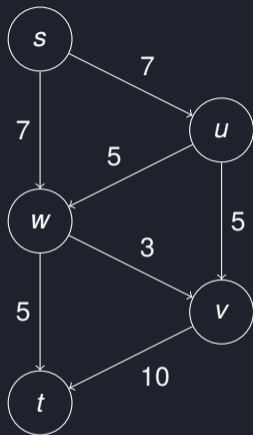


## Solution (b)



S	T	c(S,T)
{s}	{u, v, w, t}	14
{s, u}	{v, w, t}	17
{s, v}	{v, w, t}	24
{s, w}	{u, v, t}	15
{s, u, v}	{w, t}	22
{s, u, w}	{v, t}	13
{s, v, w}	{u, t}	22
{s, u, v, w}	{t}	15

## Solution (b)



S	T	$c(S,T)$
{s}	{u, v, w, t}	14
{s, u}	{v, w, t}	17
{s, v}	{v, w, t}	24
{s, w}	{u, v, t}	15
{s, u, v}	{w, t}	22
{s, u, w}	{v, t}	13
{s, v, w}	{u, t}	22
{s, u, v, w}	{t}	15

**Minimal cut:**  $S = \{s, u, w\}$ ,  $T = \{v, t\}$

**Capacity:**  $c(S, T) = 13$



*That's all Folks!*