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Exercise Sheet 2

Exercise 2.1:

Consider the following LP:

$$\begin{aligned} \textbf{maximize} \quad & x_1 + x_2 \\ \textbf{subject to} \quad & 4x_1 - x_2 \leq 8 \\ & 2x_1 + x_2 \leq 10 \\ & 5x_1 - 2x_2 \geq -2 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

a) Find all extremal points of this LP.

b) Transform the given LP into normal form and find the admissible basic points.

c) Solve the LP by evaluating the target function for each extremal point identified in (a).

Exercise 2.2:

Solve the following linear program using the Simplex algorithm from the lecture.

$$\begin{aligned} \textbf{maximize} \quad & 1x_1 + 1x_2 \\ \textbf{subject to} \quad & 1x_1 \leq 9 \\ & 1x_2 \leq 6 \\ & 1x_1 + 2x_2 \leq 16 \\ & 2x_1 + 1x_2 \leq 20 \\ & x_1 \ , \ x_2 \geq 0 \end{aligned}$$

Exercise 2.3:

Consider the following LP:

$$\begin{aligned} \textbf{maximize} \quad & 3x_1 + x_2 + 2x_3 \\ \textbf{subject to} \quad & x_1 + x_2 + 3x_3 \leq 30 \\ & 2x_1 + 2x_2 + 5x_3 \leq 24 \\ & 4x_1 + x_2 + 2x_3 \leq 36 \\ & x_1,\ x_2,\ x_3 \geq 0 \end{aligned}$$

a) Determine the initial Simplex tableau and highlight all pivot positions.

b) Solve the LP using the Simplex method.

Exercise 2.4:

a) A farmer owns 200 acres of land, on which he plans to grow barley, wheat and potatoes. The respective details are provided in the following table:

	Potatoes	Barley	Wheat	Capacity
Cultivation costs (€/acre)	1,000	2,000	3,000	150,000 €
Days of work (d/acre)	1	3	4	160 days
Net profit (€/acre)	2,000	2,000	3,000	

Maximize the farmer's total net profit.

b) The owner of a chicken farm uses two sorts of grain (called A and B) to feed the animals. Each kind contains protein, fat, carbohydrates and additional indigestible components. On the basis of A and B the farmer now wants to create a blend which contains at least 1 kg of protein, 800 g of fat and 1.8 kg of carbs. The composition and prices per kg of each sort of grain are given in the table below:

	Grain A	Grain B
Protein (per kg)	100 g	200 g
Fat (per kg)	200 g	100 g
Carbs (per kg)	100 g	600 g
Price (per kg)	8 €	12 €

Minimize the price of the blend using the Simplex method based on the normal form of the original LP instance (see Ex. 2.2) and a *helper tableau* T^{aux}.

Exercise 2.5:

Consider the following LP instance I:

$$\begin{aligned} \textbf{maximize} \quad & -2x_1 - 3x_2 \\ \textbf{subject to} \quad & x_1 + x_2 \leq 7 \\ & x_1 - x_2 \leq -16 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

Determine $opt(I_{aux})$. Does I have any valid solutions?