Algorithmics

Spring 2020 - Tutorial #1

Alexander Moch

May 4, 2020

Exercise 1.1

What is the difference between O and o, resp. Ω and ω?

$f \in O(g)$: For some $C \in \mathbb{R}^{>0}$ there exists a $n_0 \in \mathbb{N}$, such that for all $n \in \mathbb{N}$ with $n > n_0$ it holds: $f(n) \leq C \cdot g(n)$.

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$f \in o(g)$: For all $C \in \mathbb{R}^{>0}$ there exists a $n_0 \in \mathbb{N}$, such that for all $n \in \mathbb{N}$ with $n > n_0$ it holds: $f(n) < C \cdot g(n)$.

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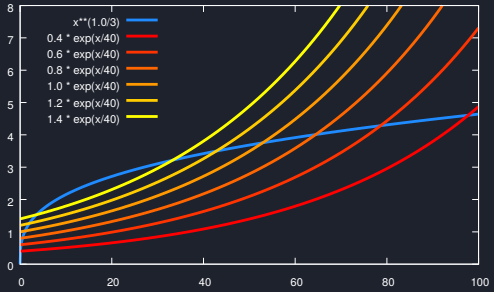
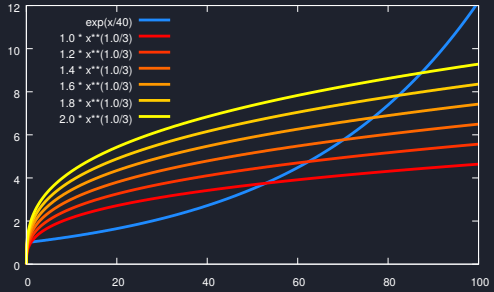
Analogously for Ω and ω.
(Replace $\leq$ by $\geq$ and $<$ by $>$)

$f \in o(g)$



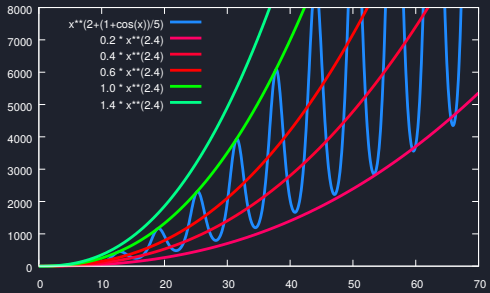
$f \in o(g)$ informal: No matter how I choose $C \in \mathbb{R}^{>0}$, from a certain point on $f(n)$ is always smaller than $C \cdot g(n)$.

$f \in \omega(g)$



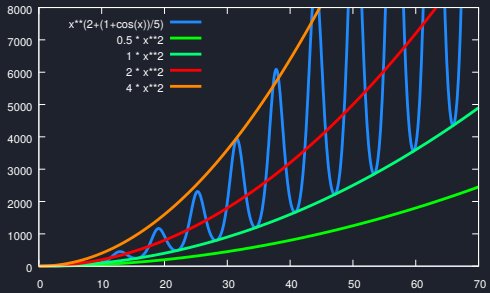
$f \in \omega(g)$ informal: No matter how I choose $C \in \mathbb{R}^{>0}$, from a certain point on $f(n)$ is always greater than $C \cdot g(n)$.

$f \in O(g)$ (in the example: $f \notin (o(g))$)



$f \in O(g)$ informal: I can choose a $C \in \mathbb{R}^{>0}$, such that from a certain point on $f(n)$ is never greater than $C \cdot g(n)$.

$f \in \Omega(g)$ (in the example: $f \notin (\omega(g))$)



$f \in \Omega(g)$ informal: I can choose a $C \in \mathbb{R}^{>0}$, such that from a certain point on $f(n)$ is never smaller than $C \cdot g(n)$.

Transitivity and Reflexivity

Transitivity

- $f(n) \in \Theta(g(n))$ and $g(n) \in \Theta(h(n))$ imply $f(n) \in \Theta(h(n))$.
- $f(n) \in O(g(n))$ and $g(n) \in O(h(n))$ imply $f(n) \in O(h(n))$.
- $f(n) \in \Omega(g(n))$ and $g(n) \in \Omega(h(n))$ imply $f(n) \in \Omega(h(n))$.
- $f(n) \in o(g(n))$ and $g(n) \in o(h(n))$ imply $f(n) \in o(h(n))$.
- $f(n) \in \omega(g(n))$ and $g(n) \in \omega(h(n))$ imply $f(n) \in \omega(h(n))$.

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- $f(n) \in \Omega(g(n))$ and $g(n) \in \Omega(h(n))$ imply $f(n) \in \Omega(h(n))$.
- $f(n) \in o(g(n))$ and $g(n) \in o(h(n))$ imply $f(n) \in o(h(n))$.
- $f(n) \in \omega(g(n))$ and $g(n) \in \omega(h(n))$ imply $f(n) \in \omega(h(n))$.

Reflexivity

- $f(n) \in \Theta(f(n))$.
- $f(n) \in O(f(n))$.
- $f(n) \in \Omega(f(n))$.

Symmetry

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Transpose Symmetry

- $f(n) \in O(g(n))$ if and only if $g(n) \in \Omega(f(n))$.
- $f(n) \in o(g(n))$ if and only if $g(n) \in \omega(f(n))$.

Exercise 1.1

Complete the following table with the symbols $O, o, \Omega, \omega, \Theta$.

	$\log n$	$2^{n/2}$	$\sqrt{n}$	5	2^n	$1/n$	n	e^n	n^2
$\log n$							o		
$2^{n/2}$									
$\sqrt{n}$									
5									
2^n									
$1/n$									
n									
e^n									
n^2									

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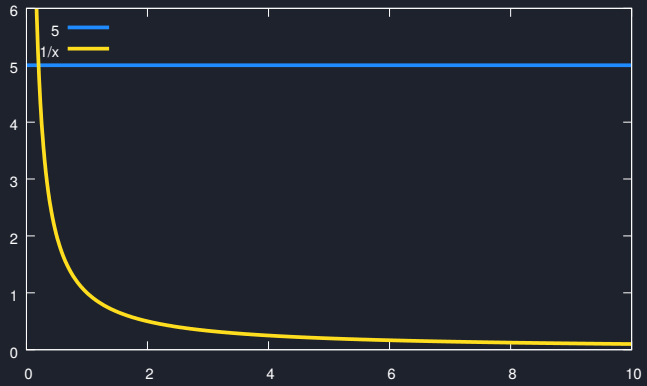
$\log n$	Θ						o		
$2^{n/2}$		Θ							
$\sqrt{n}$			Θ						
5				Θ					
2^n					Θ				
$1/n$						Θ			
n							Θ		
e^n								Θ	
n^2									Θ

Using reflexivity.

5 vs. $\frac{1}{n}$



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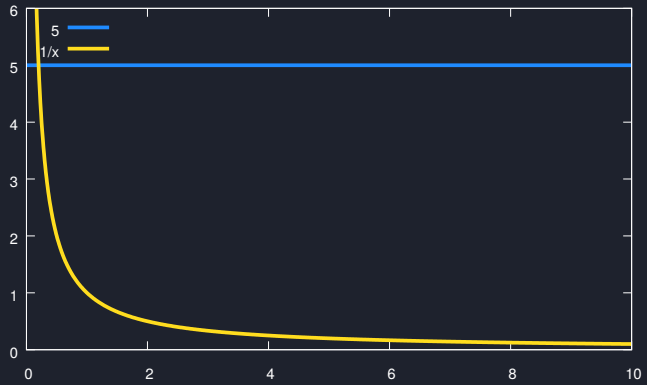
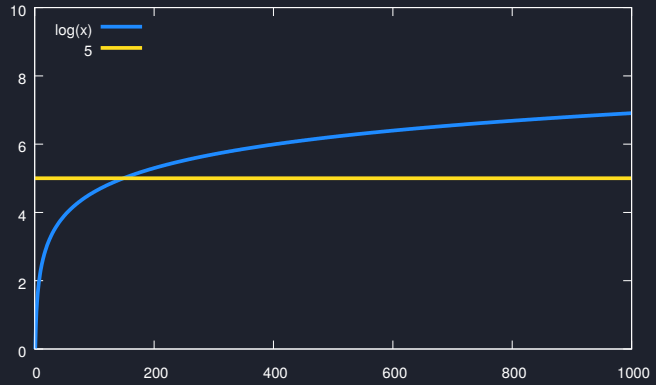
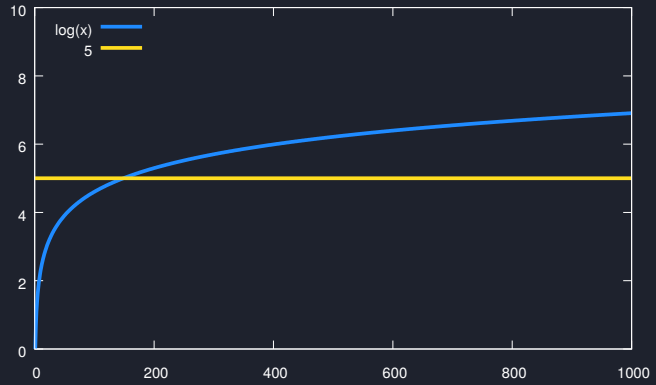


$5 \in \omega(\frac{1}{n})$, $\frac{1}{n} \in o(5)$

$\log n$ vs. 5

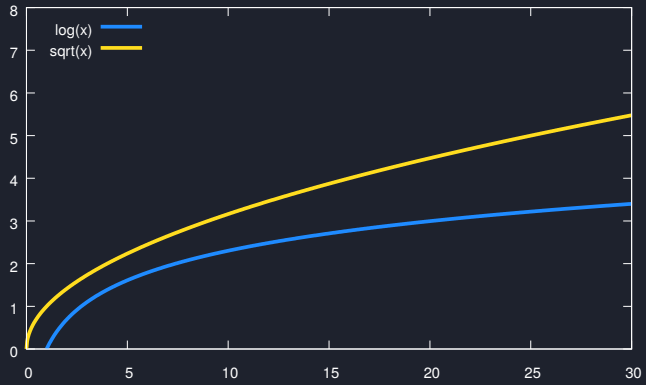


$\log n$ **vs.** 5



$\log n \in \omega(5)$, $5 \in o(\log n)$

$\log n$ vs. $\sqrt{n}$



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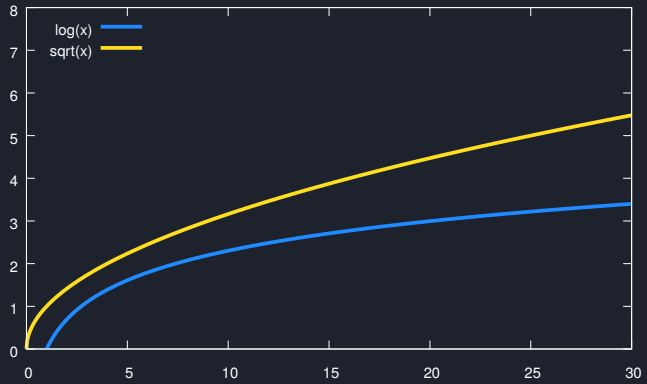
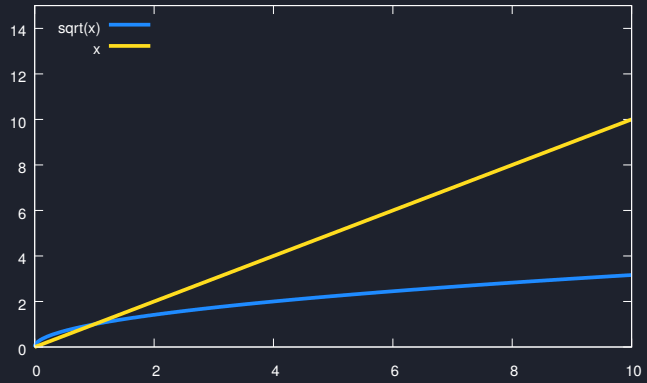
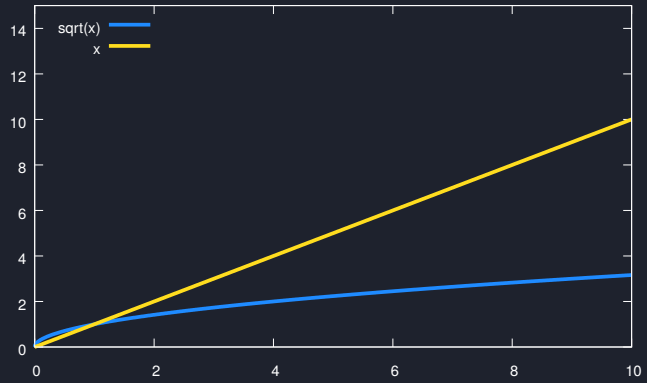


$\log n \in o(\sqrt{n})$, $\sqrt{n} \in \omega(\log n)$

$\sqrt{n}$ vs. n

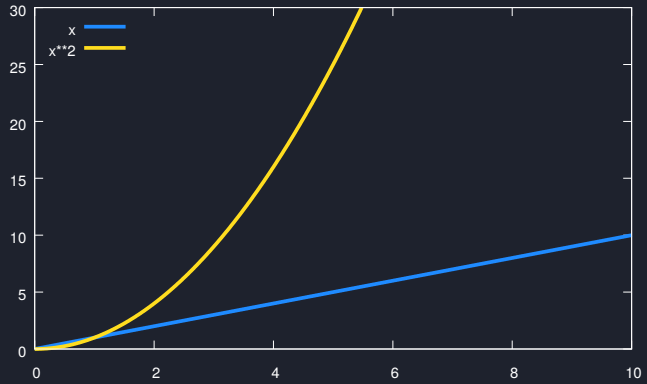


$\sqrt{n}$ vs. n

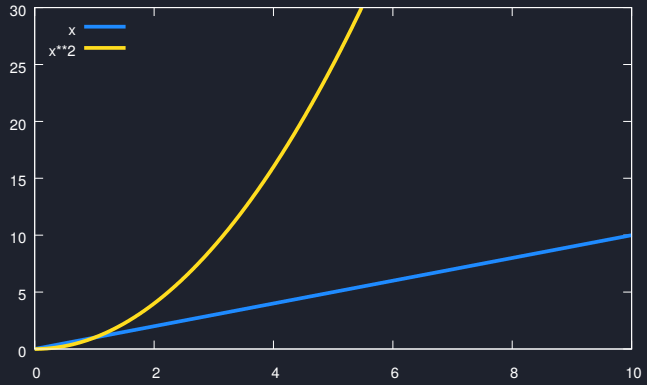


$\sqrt{n} \in o(n), n \in \omega(\sqrt{n})$

n vs. n^2

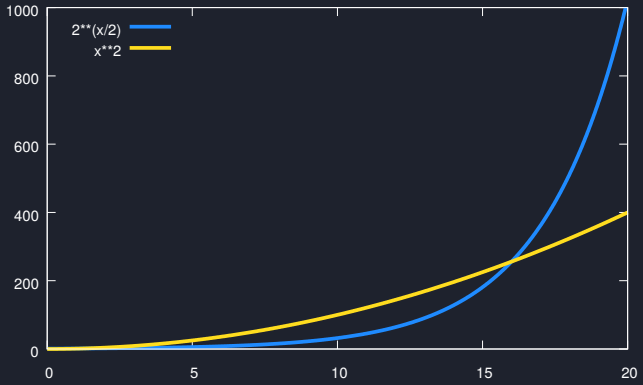


n vs. n^2



$n \in o(n^2), n^2 \in \omega(n)$

$2^{\frac{n}{2}}$ vs. n^2



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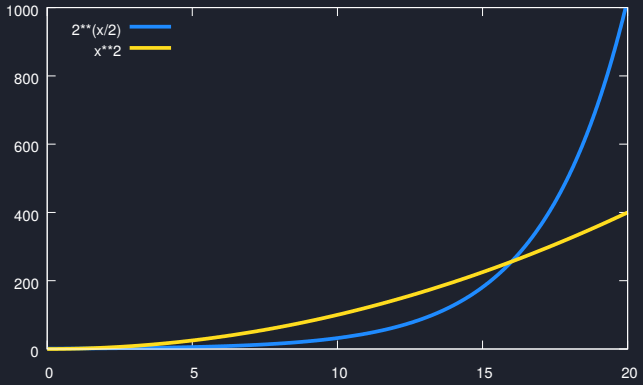
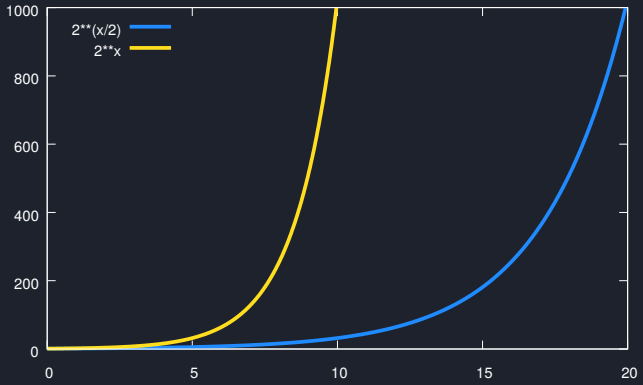
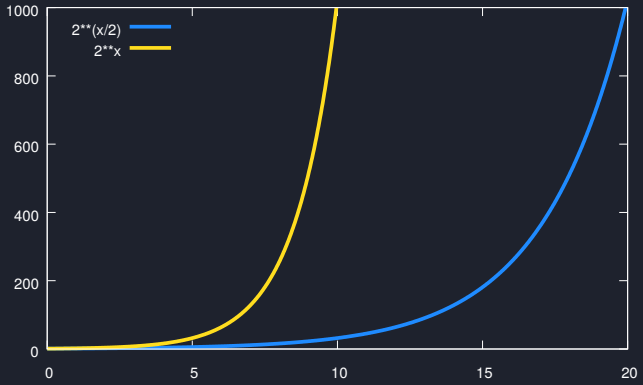


$2^{\frac{n}{2}} \in \omega(n^2), n^2 \in o(2^{\frac{n}{2}})$

$2^{\frac{n}{2}}$ vs. 2^n

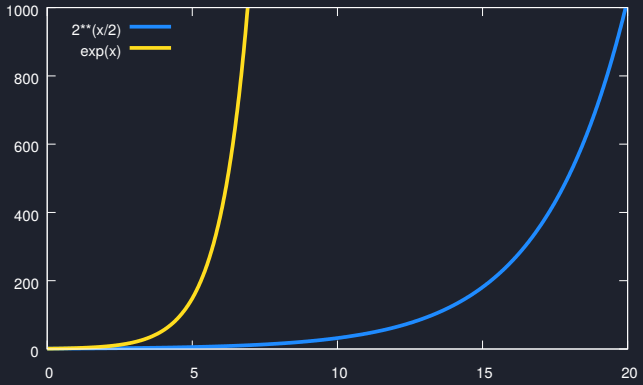


$2^{\frac{n}{2}}$ vs. 2^n

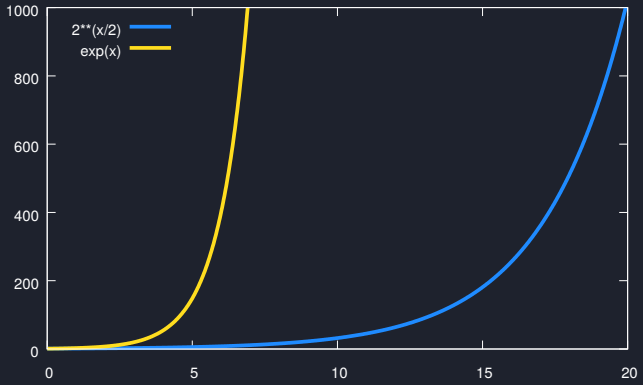


$2^{\frac{n}{2}} \in o(2^n)$, $2^n \in \omega(2^{\frac{n}{2}})$

$2^{\frac{n}{2}}$ vs. e^n



$2^{\frac{n}{2}}$ vs. e^n



$2^{\frac{n}{2}} \in o(e^n)$, $e^n \in \omega(2^{\frac{n}{2}})$

Exercise 1.1

Summary of our observations:

$$
\begin{aligned}
1/n &\in o(5) \\
5 &\in o(\log n) \\
\log n &\in o(\sqrt{n}) \\
\sqrt{n} &\in o(n) \\
n &\in o(n^2) \\
n^2 &\in o(2^{n/2}) \\
2^{n/2} &\in o(2^n) \\
2^n &\in o(e^n)
\end{aligned}
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\end{aligned}
$$

Using **transitivity** and **transpose symmetry**, that's all we need.

Exercise 1.1

Complete the following table with the symbols $O, o, \Omega, \omega, \Theta$.

	$\log n$	$2^{n/2}$	$\sqrt{n}$	5	2^n	$1/n$	n	e^n	n^2
$\log n$	Θ	o	o	ω	o	ω	o	o	o
$2^{n/2}$	ω	Θ	ω	ω	o	ω	ω	o	ω
$\sqrt{n}$	ω	o	Θ	ω	o	ω	o	o	o
5	o	o	o	Θ	o	ω	o	o	o
2^n	ω	ω	ω	ω	Θ	ω	ω	o	ω
$1/n$	o	o	o	o	o	Θ	o	o	o
n	ω	o	ω	ω	o	ω	Θ	o	o
e^n	ω	ω	ω	ω	ω	ω	ω	Θ	ω
n^2	ω	o	ω	ω	o	ω	ω	o	Θ

Exercise 1.2

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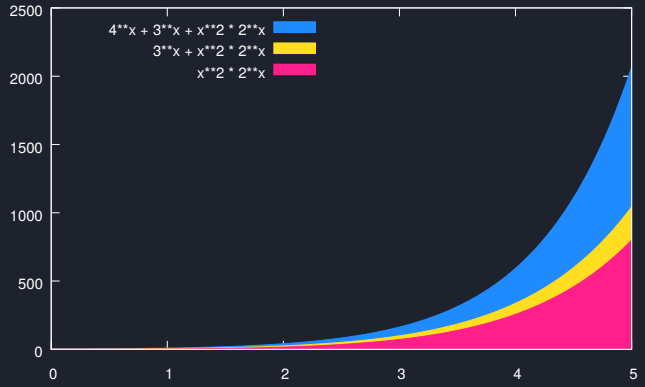
For each of the following functions f_i, find a function g_i with as few terms as possible satisfying $f_i \in \Theta(g_i)$.

$$f_1(n) = n^2 2^n + 4^n + 3^2$$
$$f_2(n) = n(n-1)/2$$
$$f_3(n) = \log n^{70}$$
$$f_4(n) = 9n \log n + 30n(\log n)^2 + n$$
$$f_5(n) = \sum_{i=1}^{n} 2^i$$
$$f_6(n) = \frac{n}{\log n} + \frac{n^2}{\log^2(n)}$$

$f_1(n) = n^2 2^n + 4^n + 3^n$



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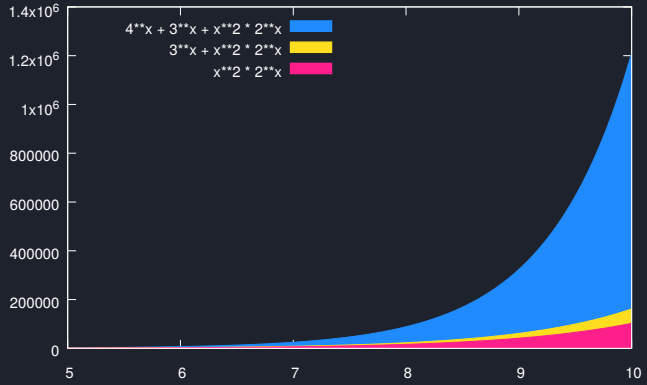

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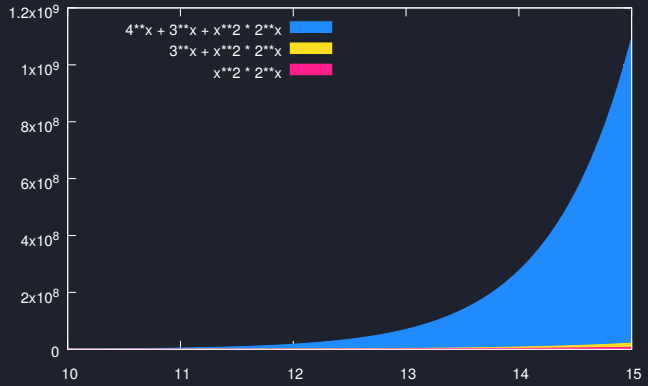
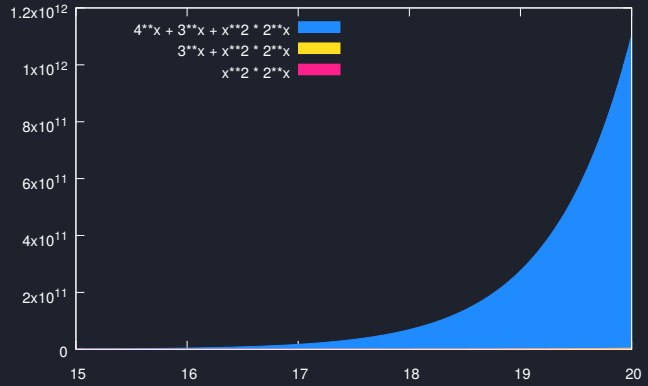
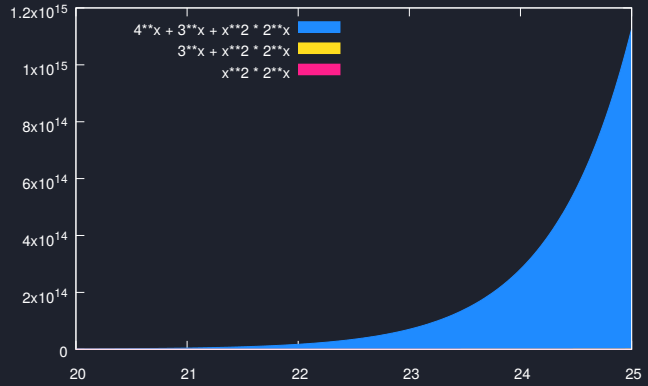


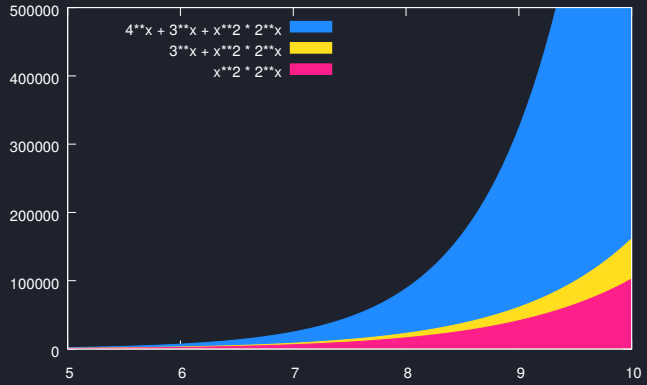
$f_1(n) = n^2 2^n + 4^n + 3^n$



$f_1(n) = n^2 2^n + 4^n + 3^n$



$f_1(n) = n^2 2^n + 4^n + 3^n$



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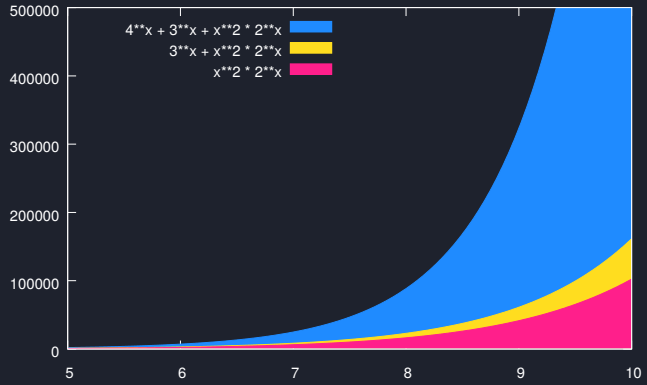
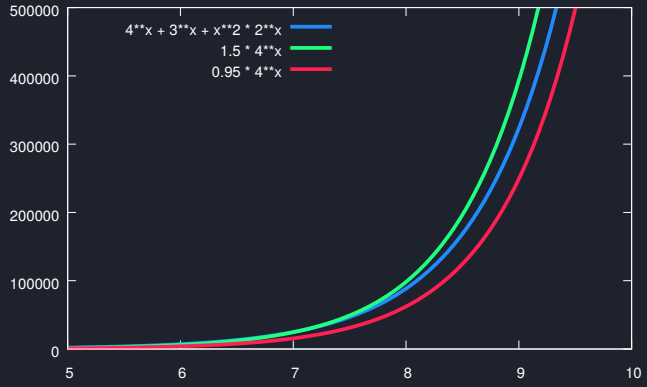


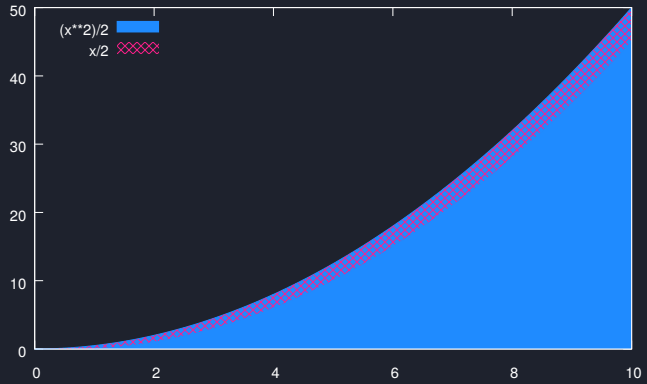
$f_1 \in \Theta(4^n)$

$f_1(n) = n^2 2^n + 4^n + 3^2$, $1.5 \cdot 4^n$, $0.95 \cdot 4^n$

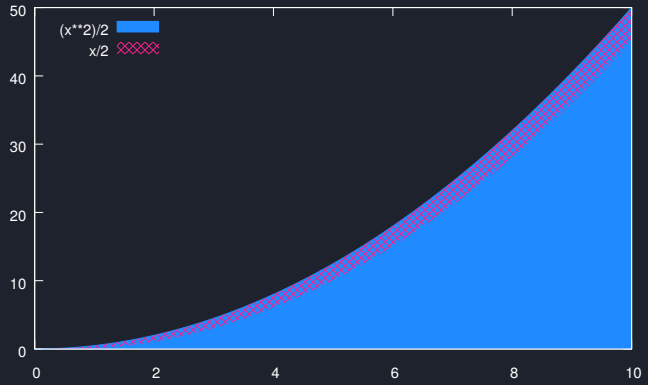


$f_1 \in \Theta(4^n)$

$$f_2(n) = n(n-1)/2 = \frac{n^2}{2} - \frac{n}{2}$$

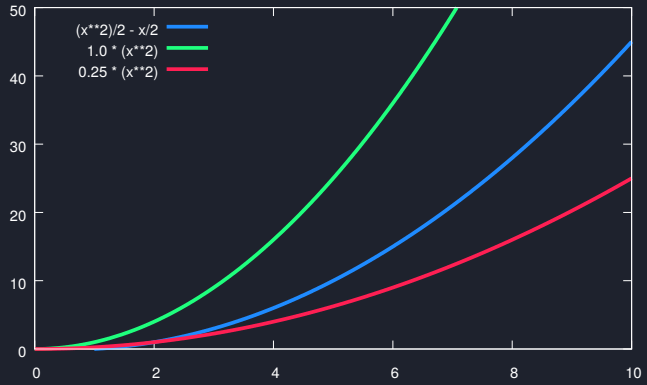


$$f_2(n) = n(n-1)/2 = \frac{n^2}{2} - \frac{n}{2}$$



$$f_2 \in \Theta(n^2)$$

$f_2(n) = n(n-1)/2$, $1.0 \cdot n^2$, $0.25 \cdot n^2$



$f_2 \in \Theta(n^2)$

$$f_3(n) = \log n^{70} = 70 \cdot \log n$$

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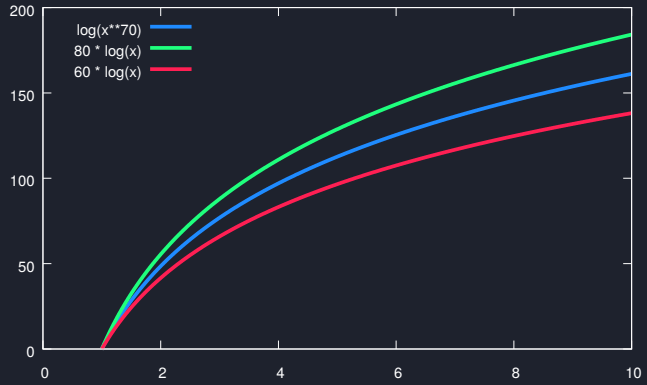
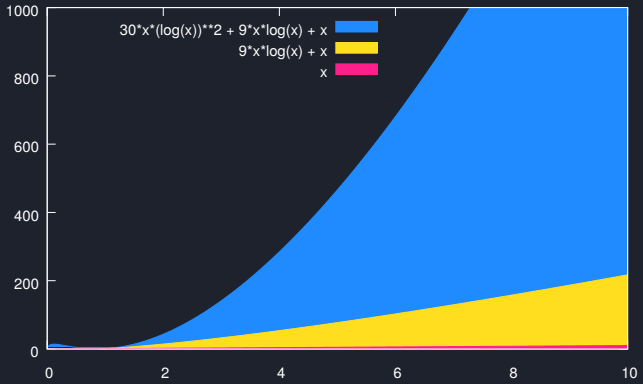
$$f_3 \in \Theta(\log n)$$

$f_3(n) = \log n^{70}$, $80 \cdot \log n$, $60 \cdot \log n$

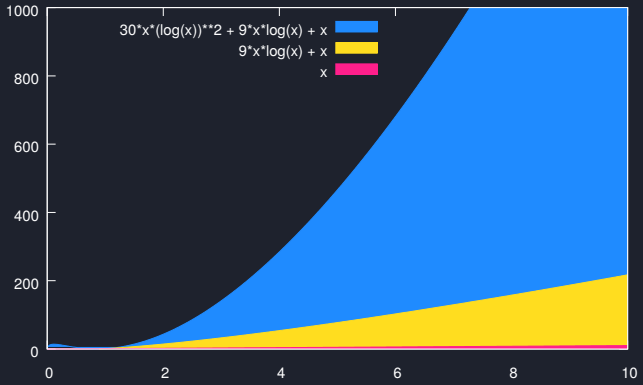


$f_3 \in \Theta(\log n)$

$f_4(n) = 9n \log n + 30n(\log n)^2 + n$

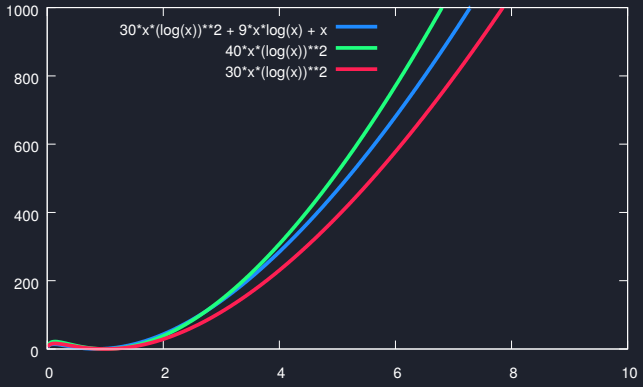


$f_4(n) = 9n \log n + 30n(\log n)^2 + n$



$f_4 \in \Theta(n(\log n)^2)$

$f_4(n) = 9n \log n + 30n(\log n)^2 + n$, $40 \cdot n(\log n)^2$, $30 \cdot n(\log n)^2$



$f_4 \in \Theta(n(\log n)^2)$

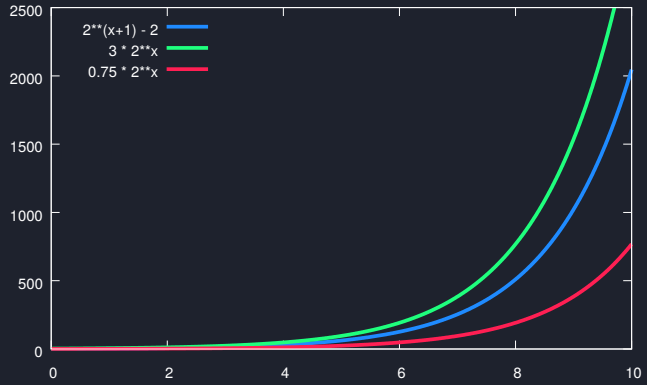
$f_5(n) = \sum_{i=1}^{n} 2^i$

$$f_5(n) = \sum_{i=1}^{n} 2^i = 2^{n+1} - 2$$

$$f_5(n) = \sum_{i=1}^{n} 2^i = 2^{n+1} - 2$$

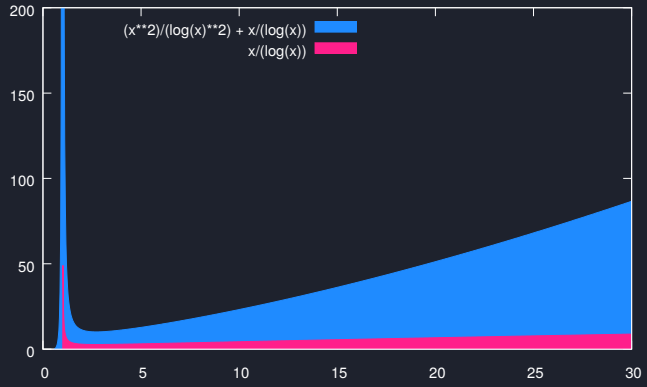
$$f_5 \in \Theta(2^n)$$

$f_5(n) = \sum_{i=1}^{n} 2^i$, $3 \cdot 2^n$, $0.75 \cdot 2^n$

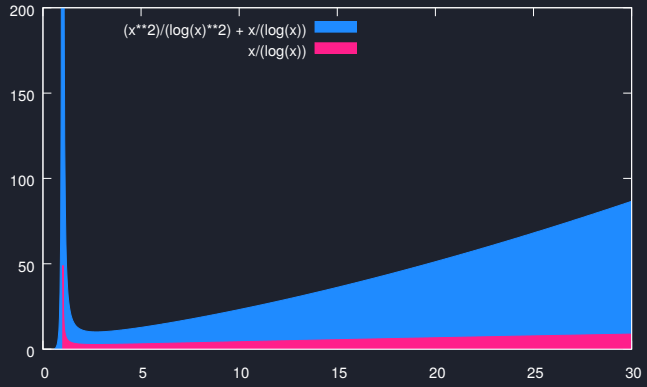


$f_5 \in \Theta(2^n)$

$$f_6(n) = \frac{n}{\log n} + \frac{n^2}{\log^2 n}$$

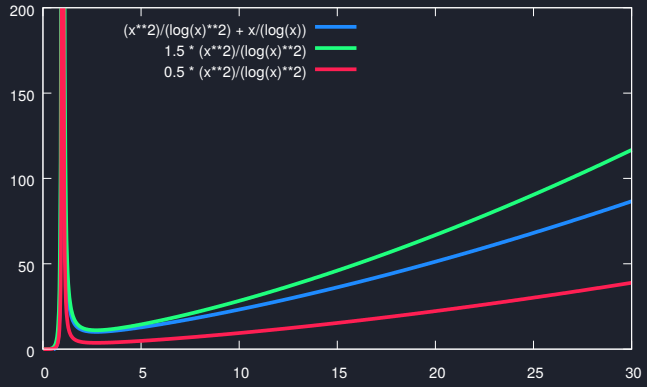


$$f_6(n) = \frac{n}{\log n} + \frac{n^2}{\log^2 n}$$



$$f_6 \in \Theta(\frac{n^2}{\log^2 n})$$

$f_6(n) = \frac{n}{\log n} + \frac{n^2}{\log^2 n}, 1.5 \cdot \frac{n^2}{\log^2 n}, 0.5 \cdot \frac{n^2}{\log^2 n}$

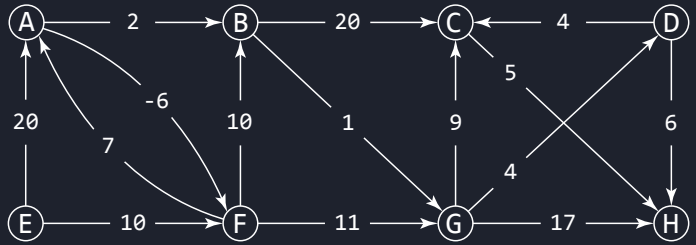


$f_6 \in \Theta(\frac{n^2}{\log^2 n})$

Exercise 1.3

Exercise 1.3

We will now consider the following graph:



Find the shortest path from vertex *E* to vertex *H* using the Bellman-Ford algorithm based on the lexicographical edge order:

Initialize and Relax

Algorithm: Initialize

```
1 A := ∅
2 for v ∈ V do
3     v.d := ∞
4     v.π := NULL
5 s.d := 0
```

Algorithm: Relax

```
1 A := ∅
2 if v.d > u.d + w(u, v) then
3     v.d := u.d + w(u, v)
4     v.π := u
```

Bellman-Ford Algorithm

Algorithm: Bellman-Ford

```
1 Initialize(G,s)
2 for 1 : |V| − 1 do
3     for (u, v) ∈ E do
4         Relax(u, v, w)
5 for (u, v) ∈ E do
6     if v.d > u.d + w(u, v) then
7         return false
8 return true
```

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-								
B	∞	-								
C	∞	-								
D	∞	-								
E	0	-								
F	∞	-								
G	∞	-								
H	∞	-								

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-								
B	∞	-								
C	∞	-								
D	∞	-								
E	0	-								
F	∞	-								
G	∞	-								
H	∞	-								

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-								
B	∞	-								
C	∞	-								
D	∞	-								
E	0	-								
F	∞	-								
G	∞	-								
H	∞	-								

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-								
B	∞	-								
C	∞	-								
D	∞	-								
E	0	-								
F	∞	-								
G	∞	-								
H	∞	-								

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-								
B	∞	-								
C	∞	-								
D	∞	-								
E	0	-								
F	∞	-								
G	∞	-								
H	∞	-								

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-								
B	∞	-								
C	∞	-								
D	∞	-								
E	0	-								
F	∞	-								
G	∞	-								
H	∞	-								

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-								
B	∞	-								
C	∞	-								
D	∞	-								
E	0	-								
F	∞	-								
G	∞	-								
H	∞	-								

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	20	E						
B	∞	-								
C	∞	-								
D	∞	-								
E	0	-								
F	∞	-								
G	∞	-								
H	∞	-								

Exercise 1.3

$$(A,B)_{2},\ (A,F)_{-6},\ (B,C)_{20},\ (B,G)_{1},\ (C,H)_{5},\ (D,C)_{4},\ (D,H)_{6},\ (E,A)_{20},$$

$$(E,F)_{10},\ (F,A)_{7},\ (F,B)_{10},\ (F,G)_{11},\ (G,C)_{9},\ (G,D)_{4},\ (G,H)_{17}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	20	E						
B	∞	-								
C	∞	-								
D	∞	-								
E	0	-								
F	∞	-	10	E						
G	∞	-								
H	∞	-								

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-								
C	∞	-								
D	∞	-								
E	0	-								
F	∞	-	10	E						
G	∞	-								
H	∞	-								

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F						
C	∞	-								
D	∞	-								
E	0	-								
F	∞	-	10	E						
G	∞	-								
H	∞	-								

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F						
C	∞	-								
D	∞	-								
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F						
H	∞	-								

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F						
C	∞	-	30	G						
D	∞	-								
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F						
H	∞	-								

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F						
C	∞	-	30	G						
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F						
H	∞	-								

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F						
C	∞	-	30	G						
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F						
H	∞	-	38	G						

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G						
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F						
H	∞	-	38	G						

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G						
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F						
H	∞	-	38	G						

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G						
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F						
H	∞	-	38	G						

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G						
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G						

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G						
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	35	C				

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	35	C				

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G						
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D				
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D				

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)},\ \underset{-6}{(A,F)},\ \underset{20}{(B,C)},\ \underset{1}{(B,G)},\ \underset{5}{(C,H)},\ \underset{4}{(D,C)},\ \underset{6}{(D,H)},\ \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)},\ \underset{7}{(F,A)},\ \underset{10}{(F,B)},\ \underset{11}{(F,G)},\ \underset{9}{(G,C)},\ \underset{4}{(G,D)},\ \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

$$\underset{2}{(A,B)}, \underset{-6}{(A,F)}, \underset{20}{(B,C)}, \underset{1}{(B,G)}, \underset{5}{(C,H)}, \underset{4}{(D,C)}, \underset{6}{(D,H)}, \underset{20}{(E,A)},$$

$$\underset{10}{(E,F)}, \underset{7}{(F,A)}, \underset{10}{(F,B)}, \underset{11}{(F,G)}, \underset{9}{(G,C)}, \underset{4}{(G,D)}, \underset{17}{(G,H)}.$$

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

No changes in the upcoming iterations.

Exercise 1.3

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Exercise 1.3

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Shortest path from E to H:

$$E \to F \to A \to B \to G \to D \to H$$

Exercise 1.3

	I		R_1^*		R_2^*		R_3^*		R_4^*	
	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$	$v.d$	$v.\pi$
A	∞	-	17	F						
B	∞	-	20	F	19	A				
C	∞	-	30	G	29	D	28	D		
D	∞	-	25	G	24	G				
E	0	-								
F	∞	-	10	E						
G	∞	-	21	F	20	B				
H	∞	-	38	G	31	D	30	D		

Shortest path from E to H:

$$E \rightarrow F \rightarrow A \rightarrow B \rightarrow G \rightarrow D \rightarrow H$$

Distance from E to H:

30

Exercise 1.4

Exercise 1.4

Blackboard only.

Exercise 1.5

Exercise 1.5

a) Let $M_1 \subseteq \mathbb{R}^n$ be defined as

$$M_1 = \left\{ x = (x_1, \ldots, x_n) \in \mathbb{R}^n \;\middle|\; \sum_{i=1}^{n} x_i \leq 1; x_i \geq 0 \; \forall i = 1, \ldots, n \right\}.$$

Determine all extremal points of M_1 and explain why your solution is correct.

b) Let $M_2 \subseteq \mathbb{R}^3$ be defined as

$$\begin{aligned} M_2 = \{ x = (x_1, x_2, x_3) \in \mathbb{R}^3 \mid\; & x_1 + x_2 + x_3 \leq 1, \\ & x_1 + 2x_2 + 4x_3 \leq 2, \\ & x_1 + 3x_2 + 9x_3 \leq 3, \\ & x_1 + 4x_2 + 16x_3 \leq 4 \}. \end{aligned}$$

How many extremal points does M_2 have? Prove your claim.

Exercise 1.5

a) Let $M_1 \subseteq \mathbb{R}^n$ be defined as

$$M_1 = \left\{ x = (x_1, \ldots, x_n) \in \mathbb{R}^n \;\middle|\; \sum_{i=1}^{n} x_i \leq 1; x_i \geq 0 \; \forall i = 1, \ldots, n \right\}.$$

Determine all extremal points of M_1 and explain why your solution is correct.

- There are n variables and $n + 1$ restrictions.
- n restrictions have to be fulfilled with equality.
- There is a total of $n + 1$ extremal points:
  - the all zero vector $(0, ..., 0)$,
  - n unit vectors with $x_i = 1$.

Exercise 1.5

b) Let $M_2 \subseteq \mathbb{R}^3$ be defined as

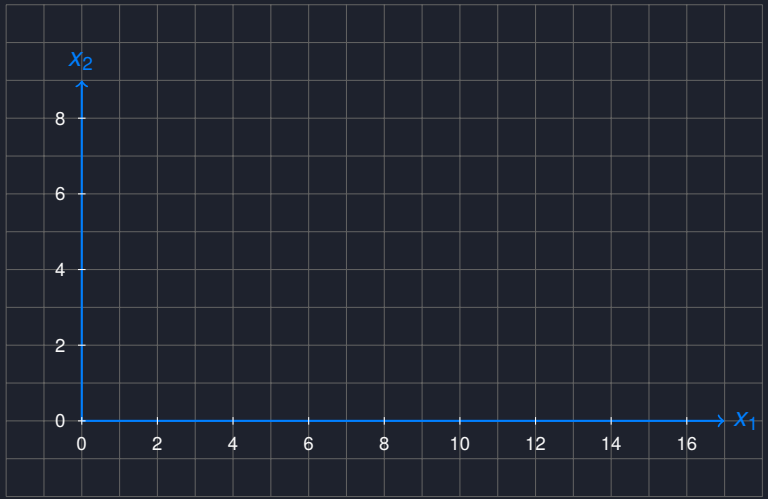
$$\begin{aligned} M_2 = \{x = (x_1, x_2, x_3) \in \mathbb{R}^3 \mid\ & x_1 + x_2 + x_3 \leq 1, \\ & x_1 + 2x_2 + 4x_3 \leq 2, \\ & x_1 + 3x_2 + 9x_3 \leq 3, \\ & x_1 + 4x_2 + 16x_3 \leq 4\}. \end{aligned}$$

How many extremal points does M_2 have? Prove your claim.

- Three restrictions have to be fulfilled with equality.
- Each subset of three restrictions is linearly independent.
- The coefficients for x_2 are equal to the last column.
- The solution is $(0, 1, 0)$.
- Due to linear independence, no other solutions exist.

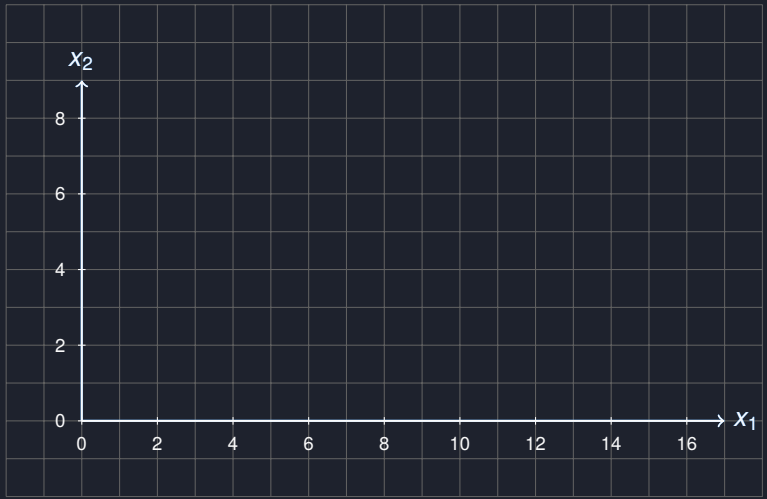
Exercise 1.6

Linear Optimization



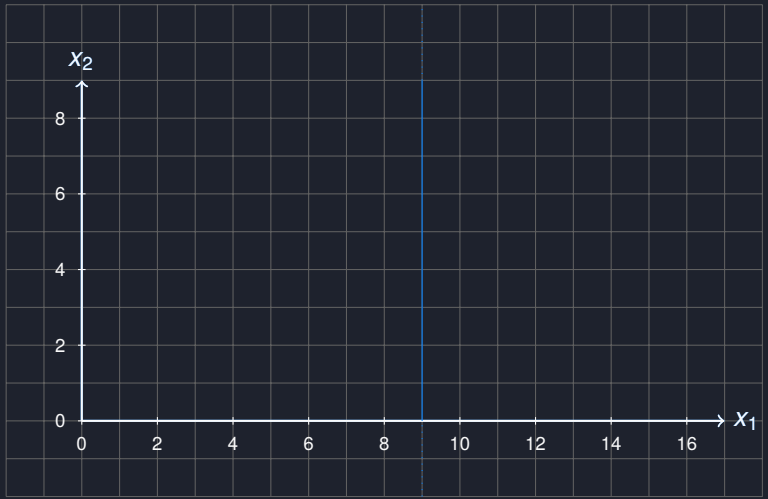
$c_0 : x_1, x_2 \geq 0$

Linear Optimization



$c_0 : x_1, x_2 \geq 0$

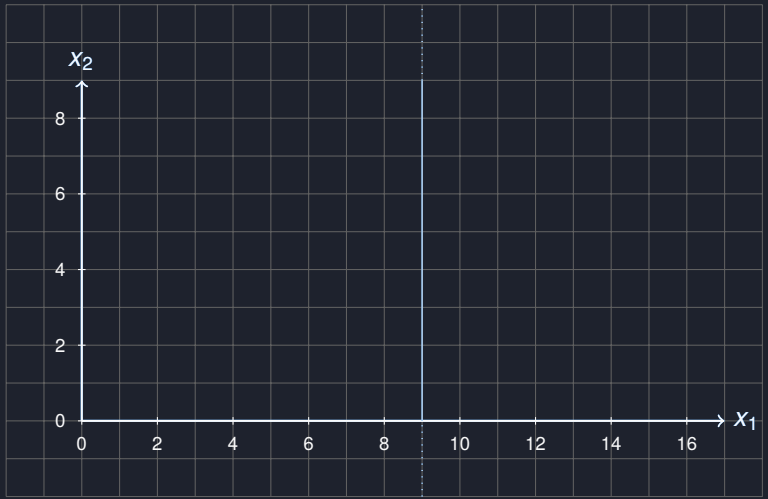
Linear Optimization



$c_0 : x_1, x_2 \geq 0$

$c_1 : x_1 \leq 9$

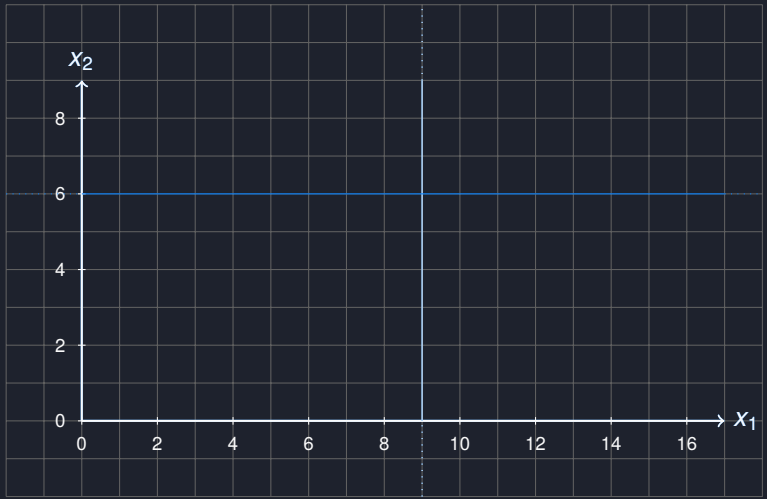
Linear Optimization



$c_0 : x_1, x_2 \geq 0$

$c_1 : x_1 \leq 9$

Linear Optimization

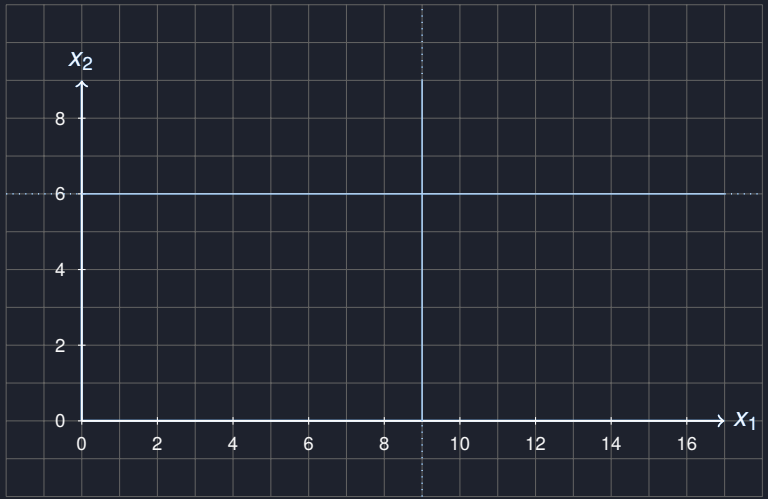


$c_0 : x_1, x_2 \geq 0$

$c_1 : x_1 \leq 9$

$c_2 : x_2 \leq 6$

Linear Optimization

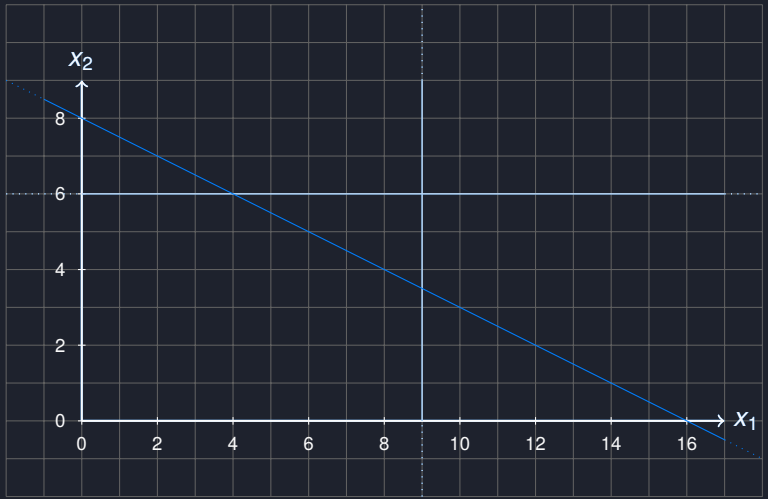


$c_0 : x_1, x_2 \geq 0$

$c_1 : x_1 \leq 9$

$c_2 : x_2 \leq 6$

Linear Optimization



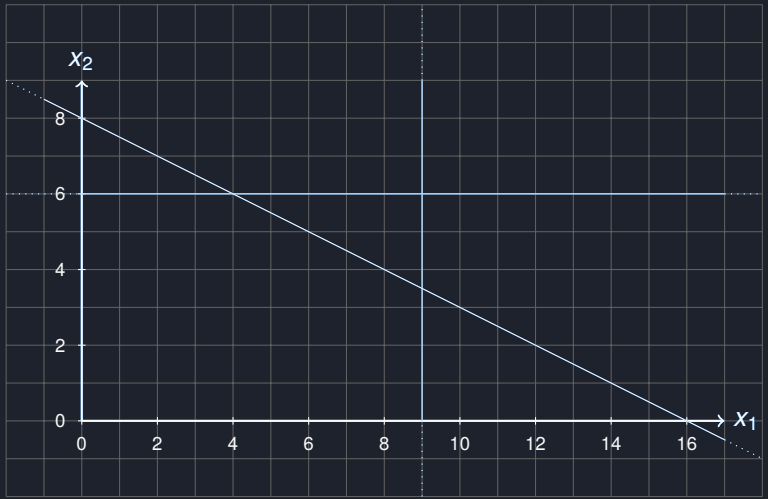
$c_0 : x_1, x_2 \geq 0$

$c_1 : x_1 \leq 9$

$c_2 : x_2 \leq 6$

$c_3 : x_1 + 2x_2 \leq 16$

Linear Optimization



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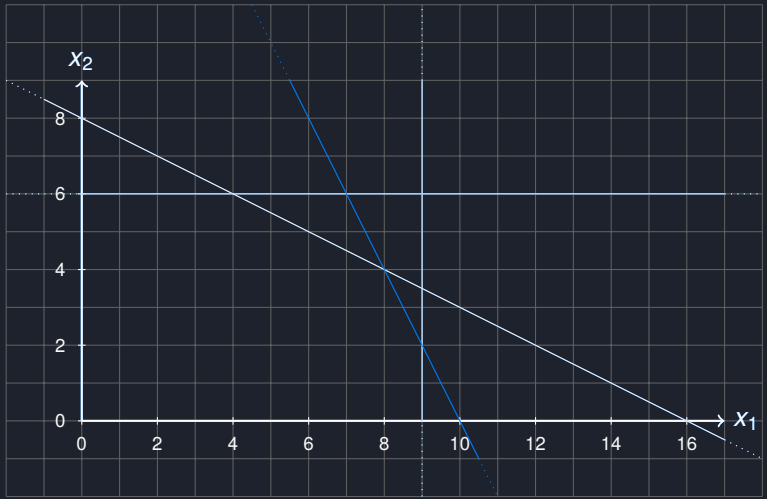
Linear Optimization



$c_0 : x_1, x_2 \geq 0$

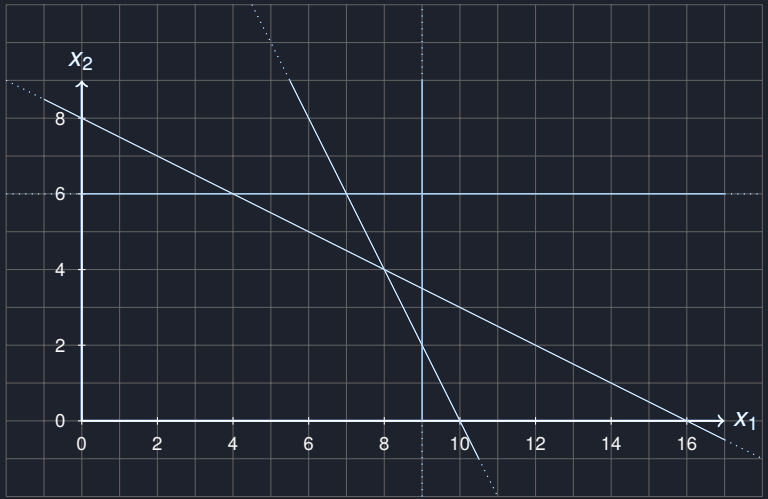
$c_1 : x_1 \leq 9$

$c_2 : x_2 \leq 6$

$c_3 : x_1 + 2x_2 \leq 16$

$c_4 : 2x_1 + x_2 \leq 20$

Linear Optimization



$c_0 : x_1, x_2 \geq 0$
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$c_2 : x_2 \leq 6$
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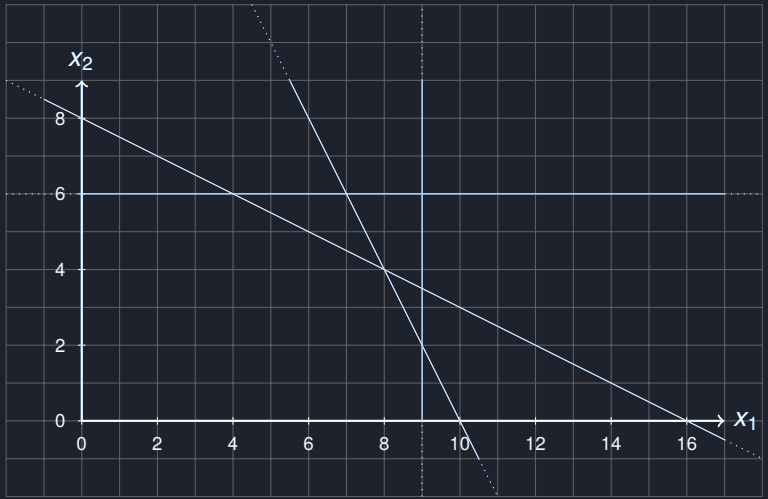
Linear Optimization



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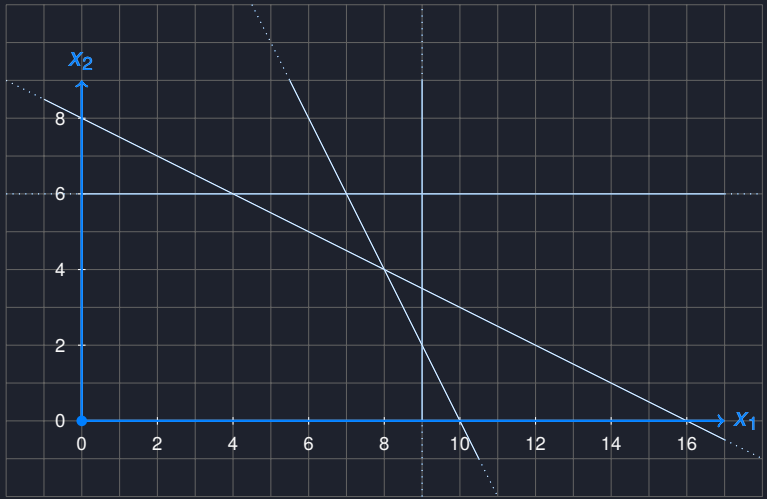
$\Rightarrow \mathrm{TF}(x_1, x_2) = x_1 + x_2$

Linear Optimization



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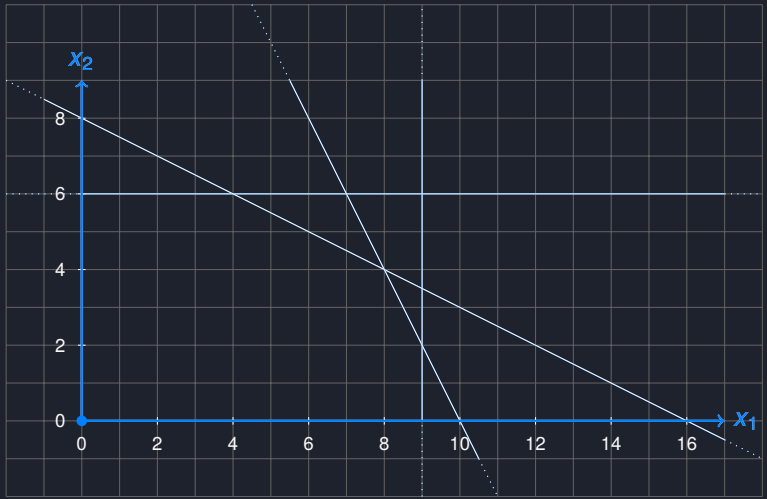
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 1:

$x_1 = x_2 \Rightarrow (0, 0)$

Linear Optimization



$c_0 : x_1, x_2 \geq 0$ ✓
$c_1 : x_1 \leq 9$ ✓
$c_2 : x_2 \leq 6$ ✓
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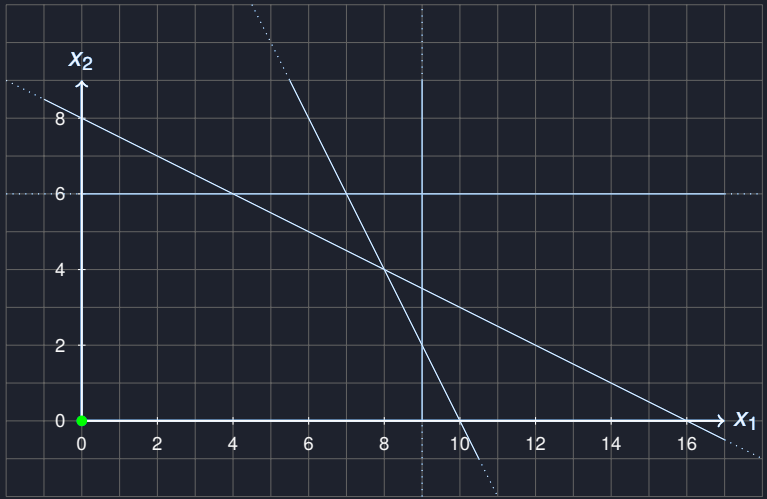
Linear Optimization



$c_0 : x_1, x_2 \geq 0$

$c_1 : x_1 \leq 9$

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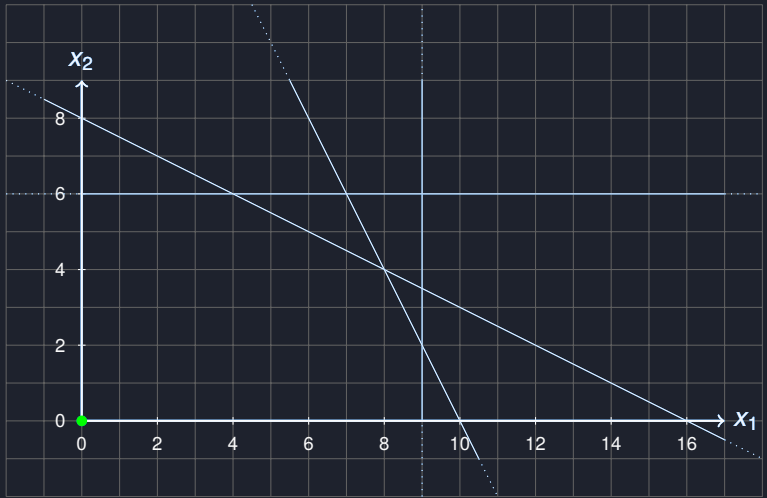
Linear Optimization



$c_0 : x_1, x_2 \geq 0$

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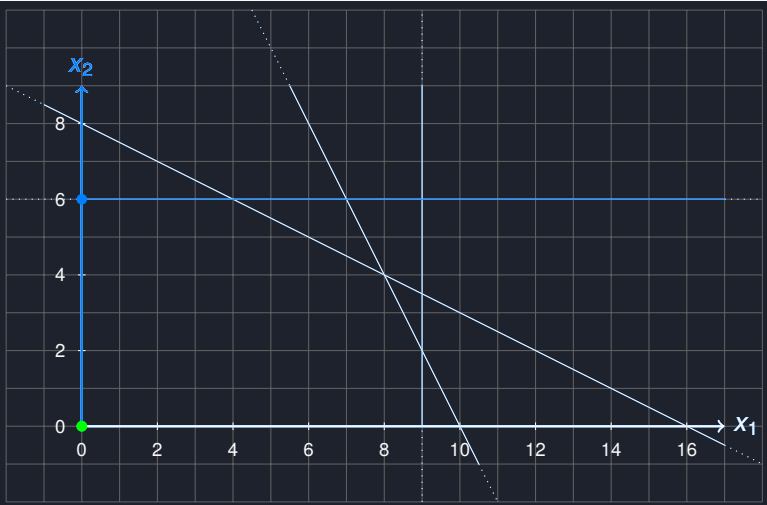
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 1:

$x_1 = x_2 \Rightarrow (0, 0)$

TF(0,0)=0+0=0

Linear Optimization

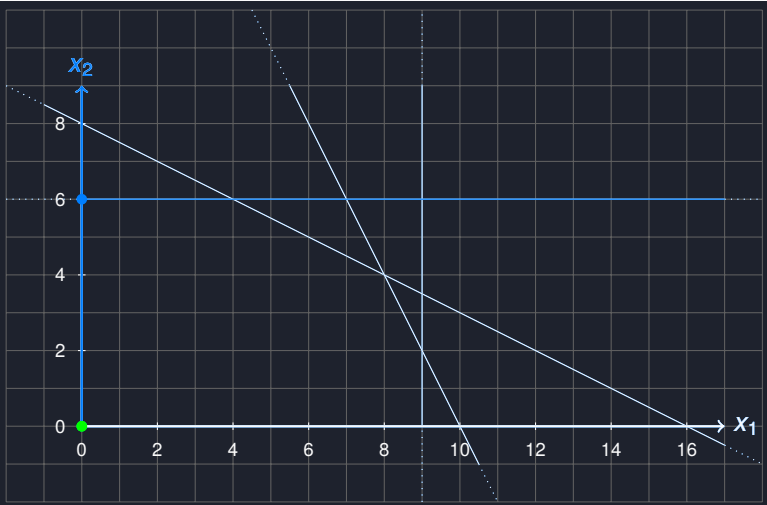


Extremal Points: TF(0,0)=0

$c_0 : x_1, x_2 \geq 0$
$c_1 : x_1 \leq 9$
$c_2 : x_2 \leq 6$
$c_3 : x_1 + 2x_2 \leq 16$
$c_4 : 2x_1 + x_2 \leq 20$
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 2:
$x_2 = c_2 \Rightarrow (0, 6)$

Linear Optimization



Extremal Points: TF(0,0)=0

$c_0 : x_1, x_2 \geq 0$ ✓
$c_1 : x_1 \leq 9$ ✓
$c_2 : x_2 \leq 6$ ✓
$c_3 : x_1 + 2x_2 \leq 16$ ✓
$c_4 : 2x_1 + x_2 \leq 20$ ✓
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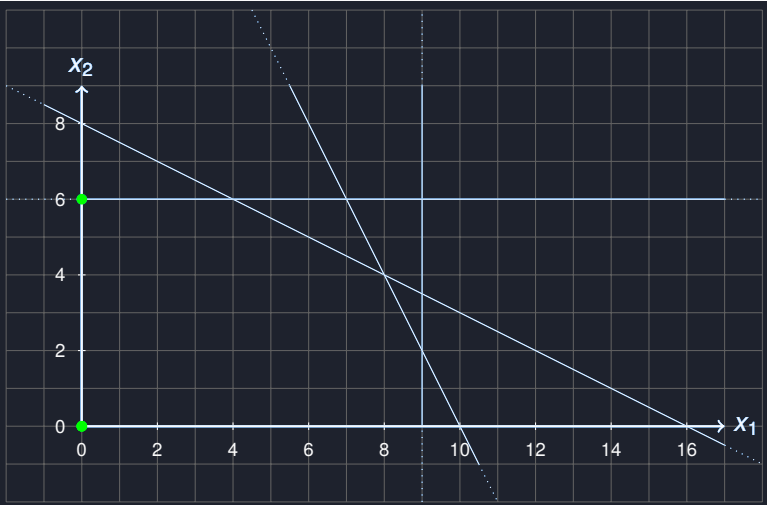
Linear Optimization



Extremal Points: TF(0,0)=0

$c_0 : x_1, x_2 \geq 0$
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$c_2 : x_2 \leq 6$
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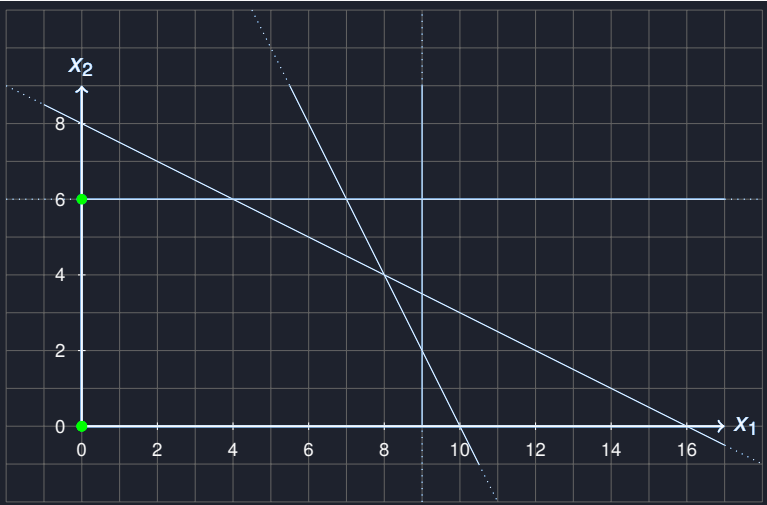
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 2:
$x_2 = c_2 \Rightarrow (0, 6)$

Linear Optimization

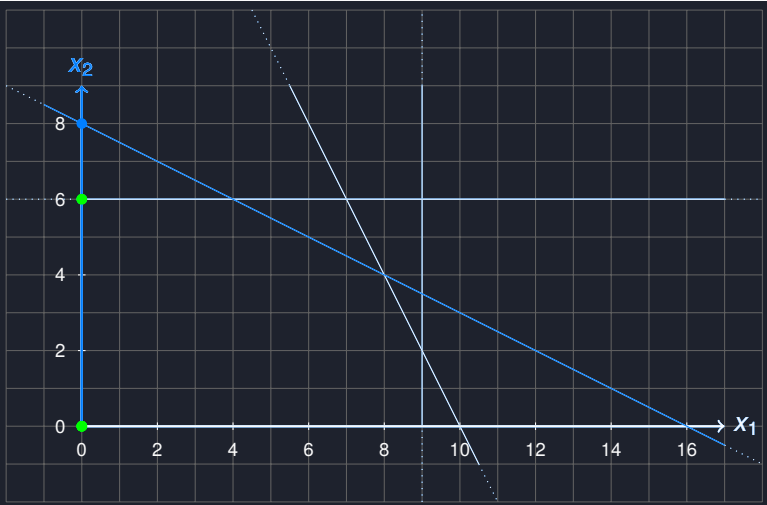


Extremal Points: TF(0,0)=0

$c_0 : x_1, x_2 \geq 0$
$c_1 : x_1 \leq 9$
$c_2 : x_2 \leq 6$
$c_3 : x_1 + 2x_2 \leq 16$
$c_4 : 2x_1 + x_2 \leq 20$
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 2:
$x_2 = c_2 \Rightarrow (0, 6)$
TF(0,6)=0+6=6

Linear Optimization

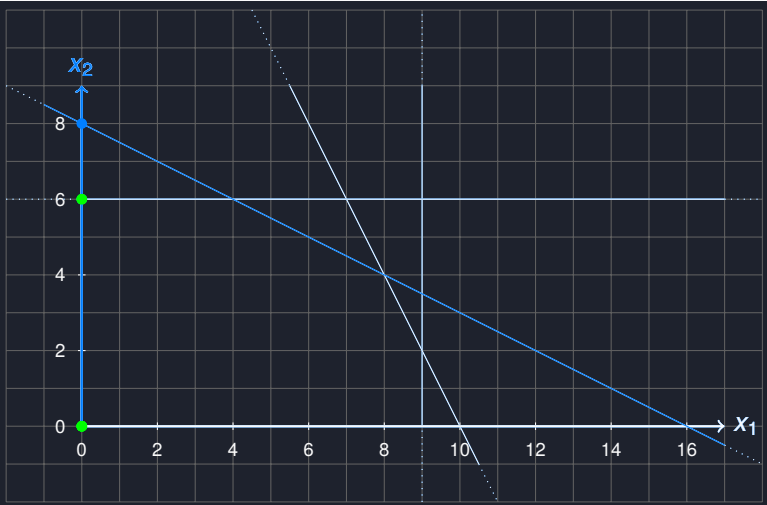


Extremal Points: TF(0,0)=0, TF(0,6)=6

$c_0 : x_1, x_2 \geq 0$
$c_1 : x_1 \leq 9$
$c_2 : x_2 \leq 6$
$c_3 : x_1 + 2x_2 \leq 16$
$c_4 : 2x_1 + x_2 \leq 20$
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 3:
$x_2 = c_3 \Rightarrow (0, 8)$

Linear Optimization

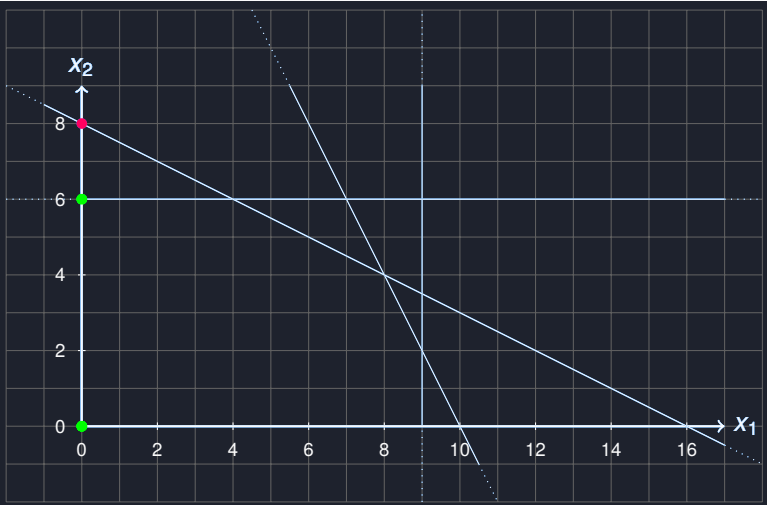


Extremal Points: TF(0,0)=0, TF(0,6)=6

$c_0 : x_1, x_2 \geq 0$ ✓
$c_1 : x_1 \leq 9$ ✓
$c_2 : x_2 \leq 6$ ✗
$c_3 : x_1 + 2x_2 \leq 16$ ✓
$c_4 : 2x_1 + x_2 \leq 20$ ✓
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 3:
$x_2 = c_3 \Rightarrow (0, 8)$

Linear Optimization



Extremal Points: TF(0,0)=0, TF(0,6)=6

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EP Candidate 3:
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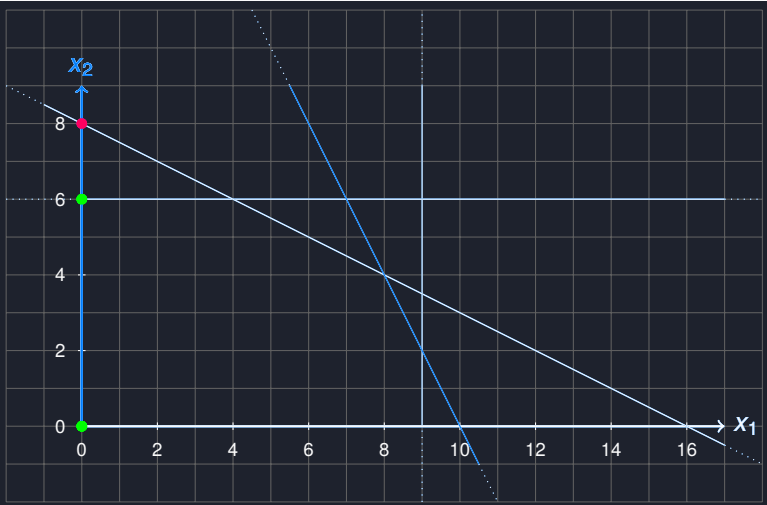
Linear Optimization



Extremal Points: TF(0,0)=0, TF(0,6)=6

$c_0 : x_1, x_2 \geq 0$

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$c_3 : x_1 + 2x_2 \leq 16$

$c_4 : 2x_1 + x_2 \leq 20$

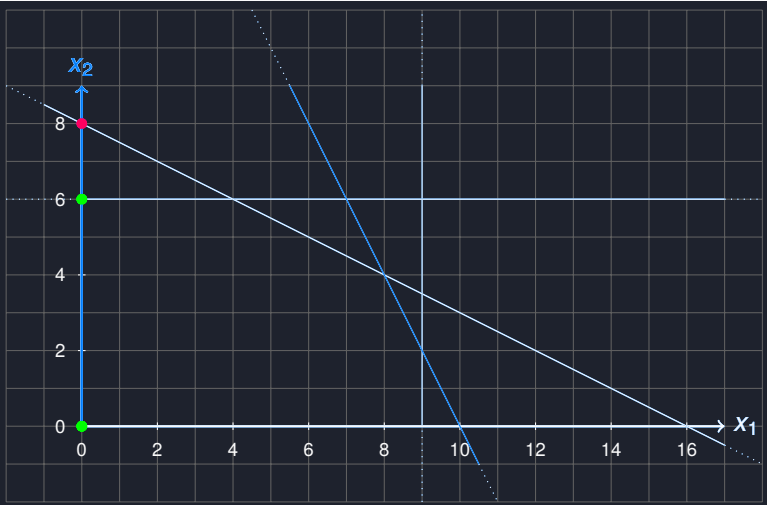
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 4:

$x_2 = c_4 \Rightarrow (0, 20)$

$\Rightarrow$ Not pictured

Linear Optimization



Extremal Points: TF(0,0)=0, TF(0,6)=6

$c_0 : x_1, x_2 \geq 0$ ✓

$c_1 : x_1 \leq 9$ ✓

$c_2 : x_2 \leq 6$ ✗

$c_3 : x_1 + 2x_2 \leq 16$ ✗

$c_4 : 2x_1 + x_2 \leq 20$ ✓

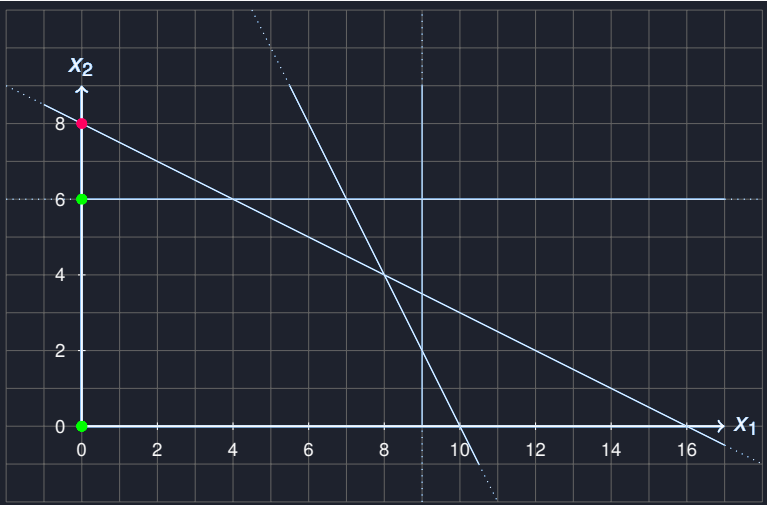
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 4:

$x_2 = c_4 \Rightarrow (0, 20)$

$\Rightarrow$ Not pictured

Linear Optimization



Extremal Points: TF(0,0)=0, TF(0,6)=6

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EP Candidate 4:

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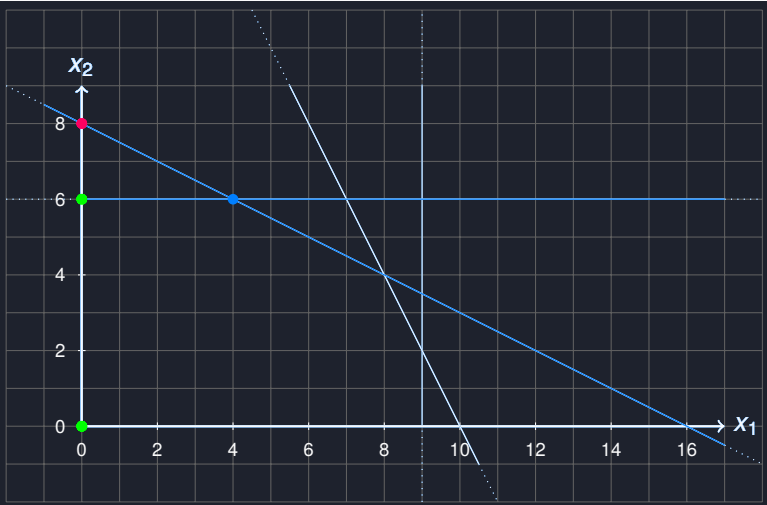
Linear Optimization



Extremal Points: TF(0,0)=0, TF(0,6)=6

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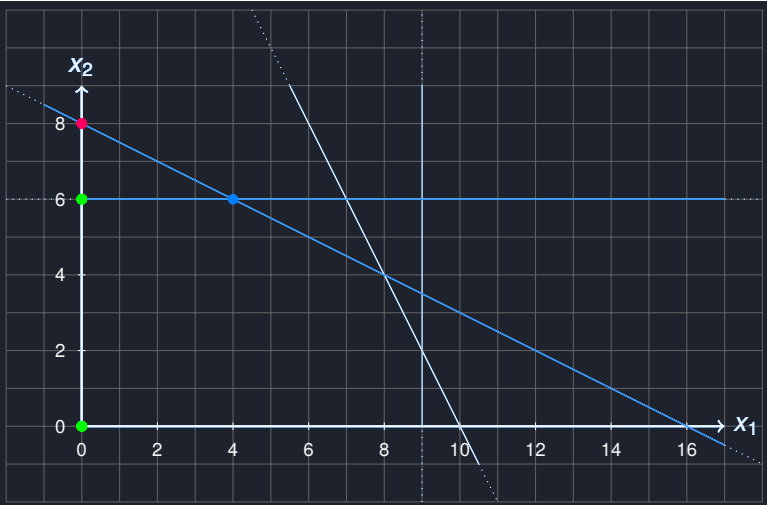
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 5:
$c_2 = c_3 \Rightarrow (4, 6)$

Linear Optimization



Extremal Points: TF(0,0)=0, TF(0,6)=6

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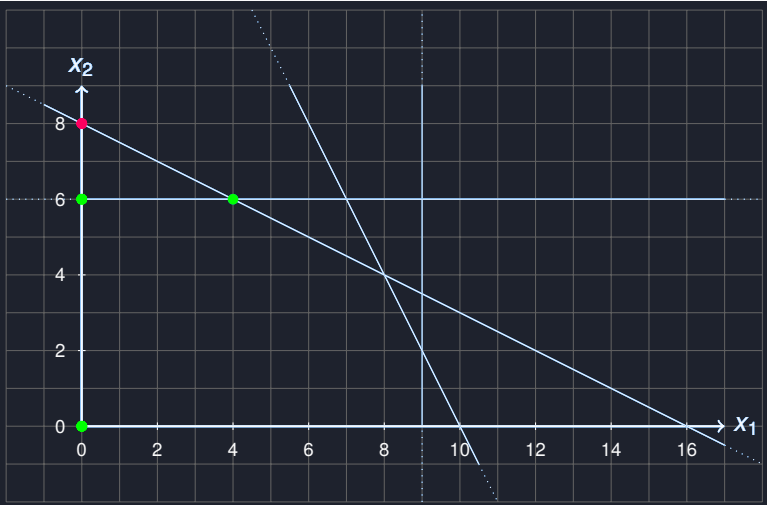
$c_4 : 2x_1 + x_2 \leq 20$ ✓
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 5:
$c_2 = c_3 \Rightarrow (4, 6)$

Linear Optimization



Extremal Points: TF(0,0)=0, TF(0,6)=6

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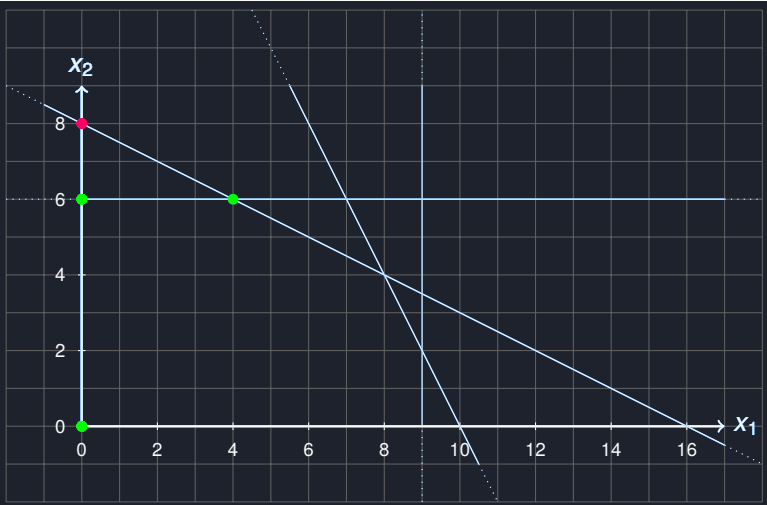
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 5:
$c_2 = c_3 \Rightarrow (4, 6)$

Linear Optimization



Extremal Points: TF(0,0)=0, TF(0,6)=6

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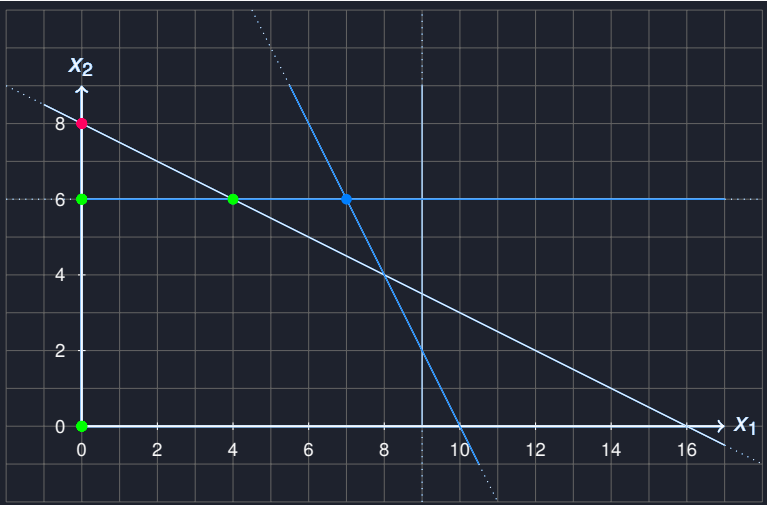
$c_4 : 2x_1 + x_2 \leq 20$
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 5:
$c_2 = c_3 \Rightarrow (4, 6)$
TF(4,6)=4+6=10

Linear Optimization



Extremal Points: TF(0,0)=0, TF(0,6)=6, TF(4,6)=10

$c_0 : x_1, x_2 \geq 0$
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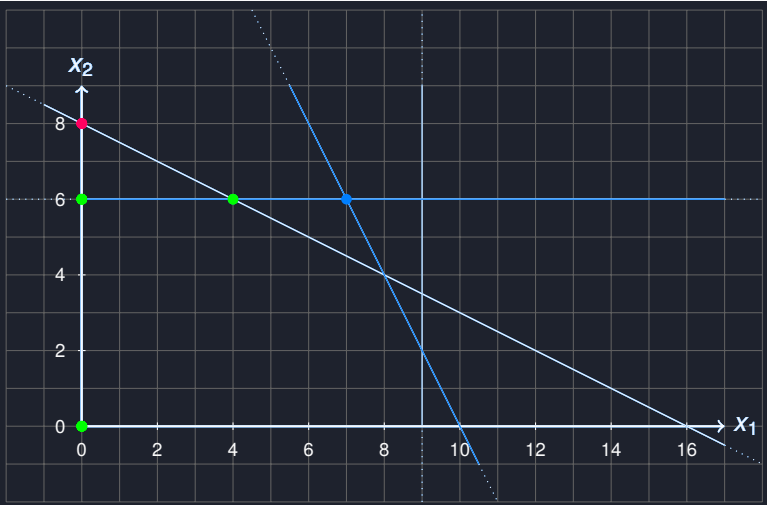
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 6:
$c_2 = c_4 \Rightarrow (7, 6)$

Linear Optimization

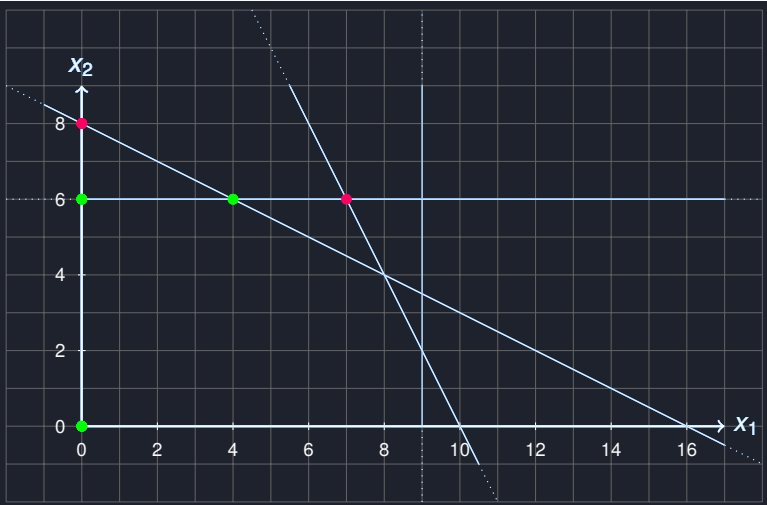


$c_0 : x_1, x_2 \geq 0$ ✓
$c_1 : x_1 \leq 9$ ✓
$c_2 : x_2 \leq 6$ ✓
$c_3 : x_1 + 2x_2 \leq 16$ ✗
$c_4 : 2x_1 + x_2 \leq 20$ ✓
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 6:
$c_2 = c_4 \Rightarrow (7, 6)$

Extremal Points: TF(0,0)=0, TF(0,6)=6, TF(4,6)=10

Linear Optimization



Extremal Points: TF(0,0)=0, TF(0,6)=6, TF(4,6)=10

$c_0 : x_1, x_2 \geq 0$
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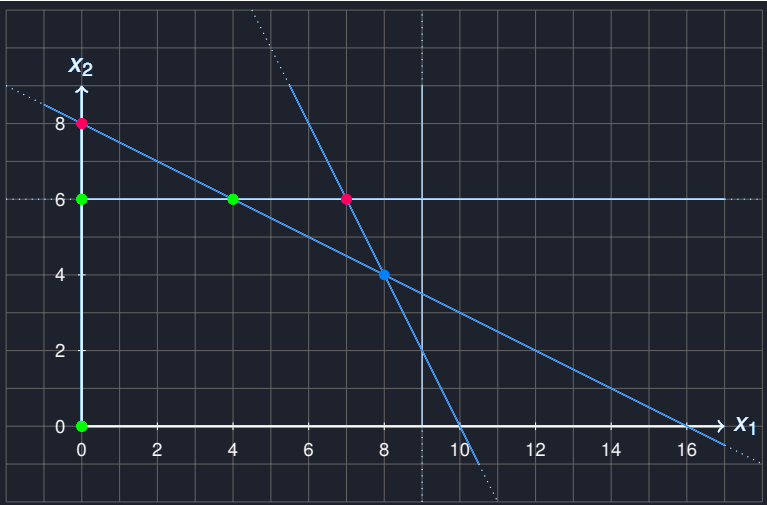
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 6:
$c_2 = c_4 \Rightarrow (7, 6)$

Linear Optimization



Extremal Points: TF(0,0)=0, TF(0,6)=6, TF(4,6)=10

$c_0 : x_1, x_2 \geq 0$
$c_1 : x_1 \leq 9$
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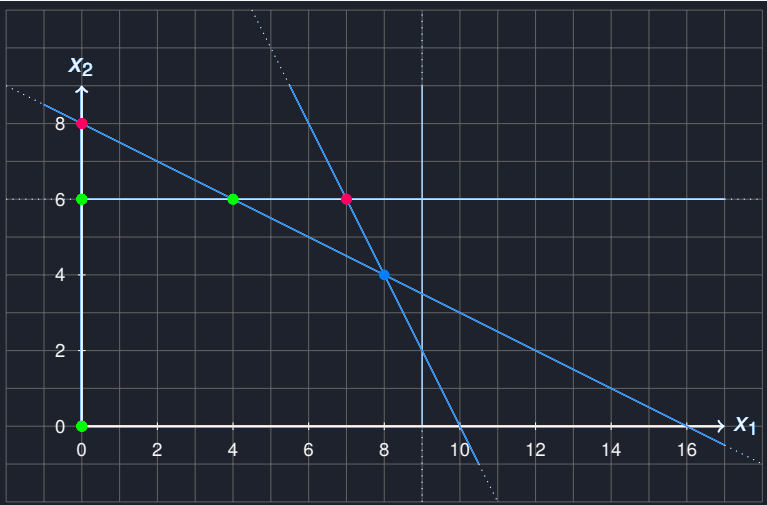
$c_3 : x_1 + 2x_2 \leq 16$
$c_4 : 2x_1 + x_2 \leq 20$
$\Rightarrow \text{TF}(x_1, x_2) = x_1 + x_2$

EP Candidate 7:
$c_3 = c_4 \Rightarrow (8, 4)$

Linear Optimization



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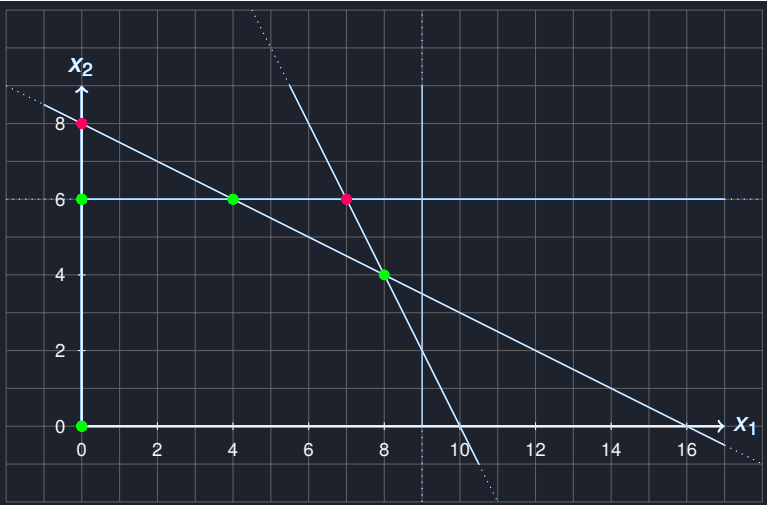
Extremal Points: TF(0,0)=0, TF(0,6)=6, TF(4,6)=10

Linear Optimization



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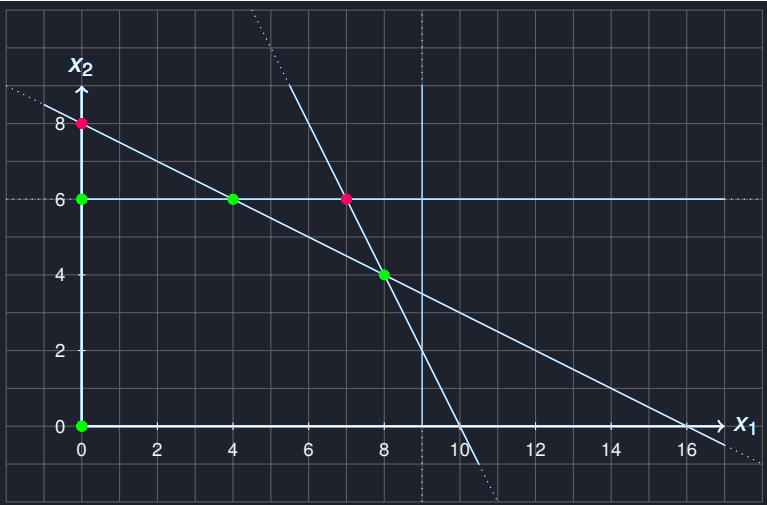
Linear Optimization



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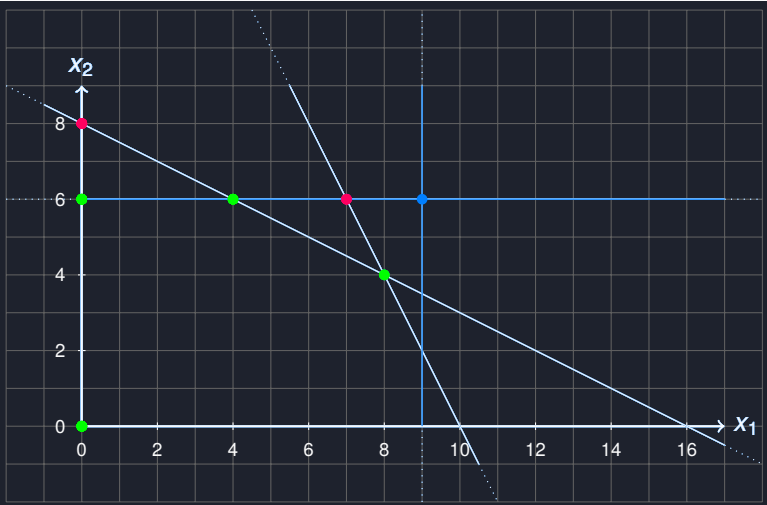
EP Candidate 7:
$c_3 = c_4 \Rightarrow (8, 4)$
TF(8,4)=8+4=12

Extremal Points: TF(0,0)=0, TF(0,6)=6, TF(4,6)=10

Linear Optimization



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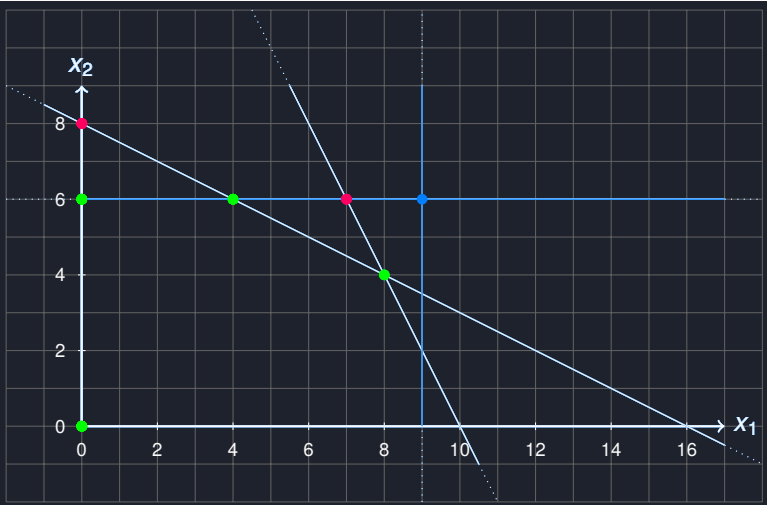
EP Candidate 8:
$c_1 = c_2 \Rightarrow (9, 6)$

Linear Optimization



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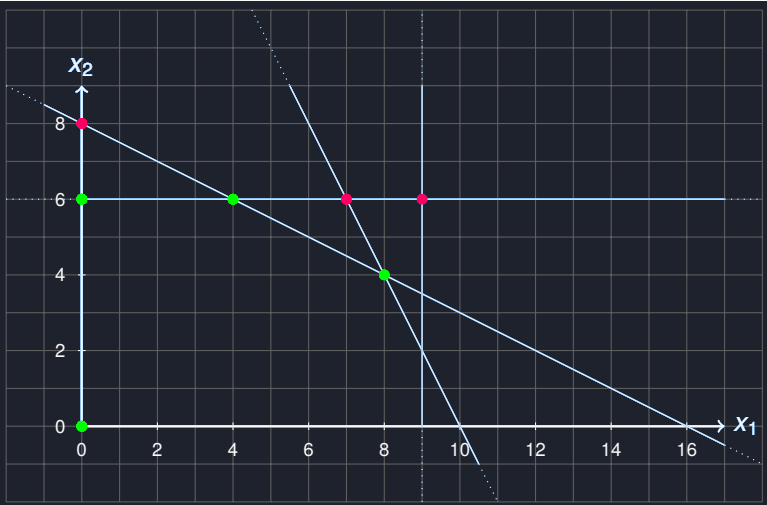

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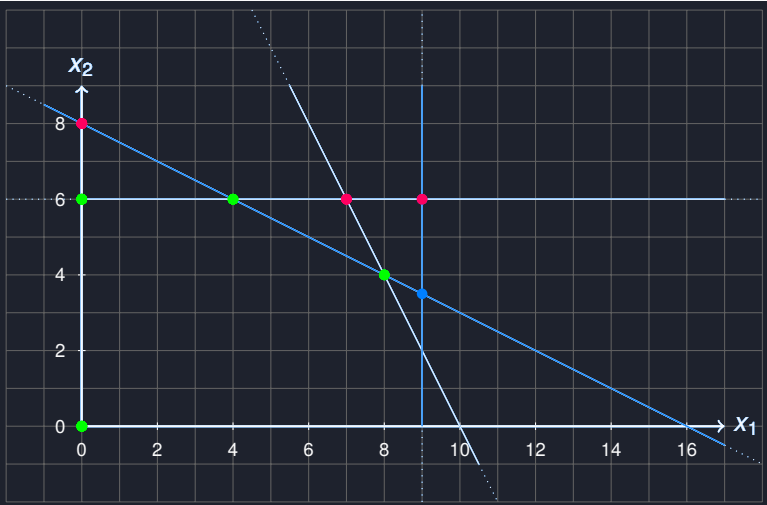

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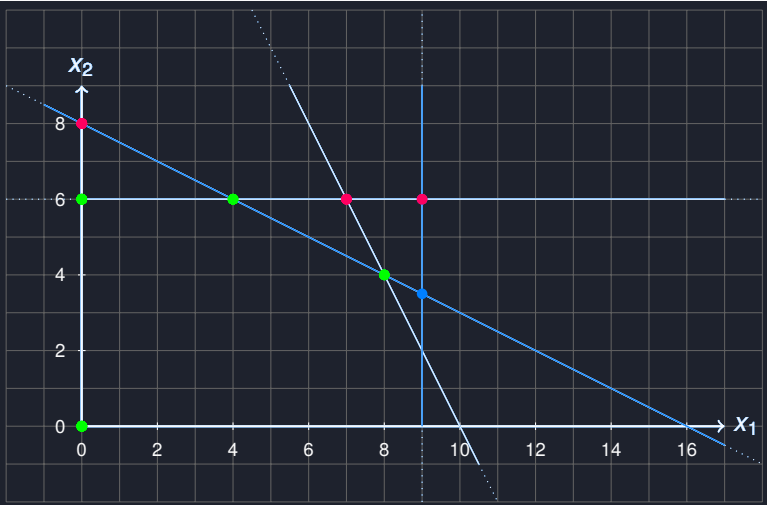
EP Candidate 9:
$c_1 = c_3 \Rightarrow (9, 3.5)$

Extremal Points: TF(0,0)=0, TF(0,6)=6, TF(4,6)=10, TF(8,4)=12

Linear Optimization



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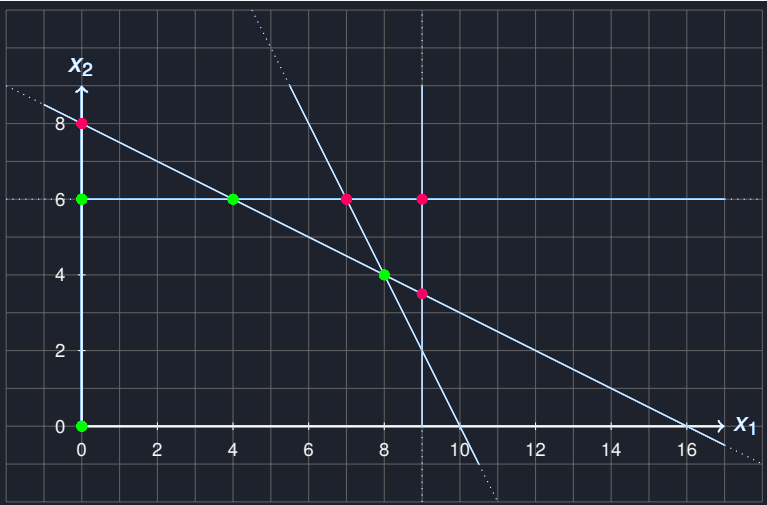

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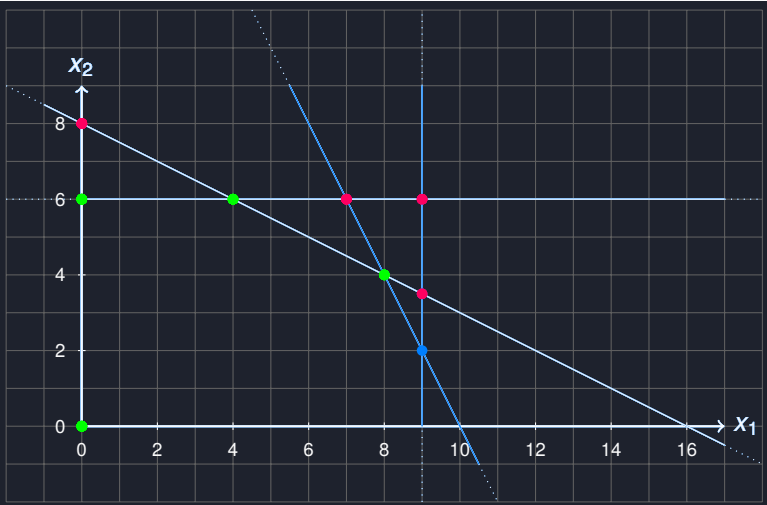

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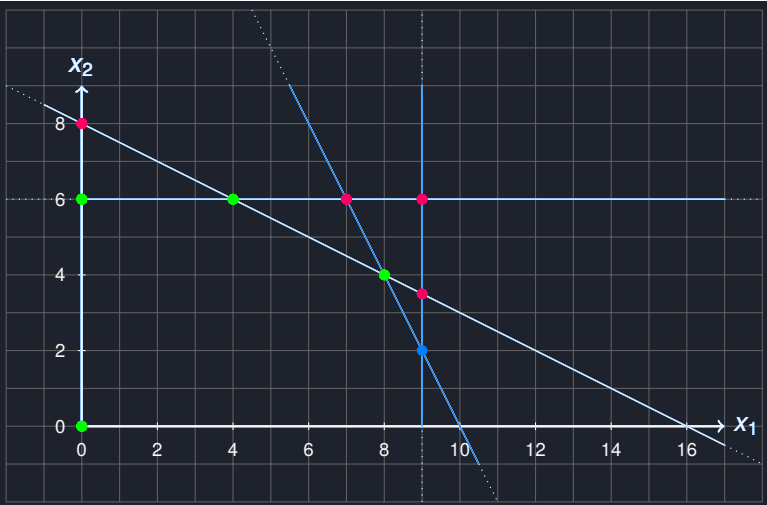
EP Candidate 10:
$c_1 = c_4 \Rightarrow (9, 2)$

Linear Optimization



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EP Candidate 10:
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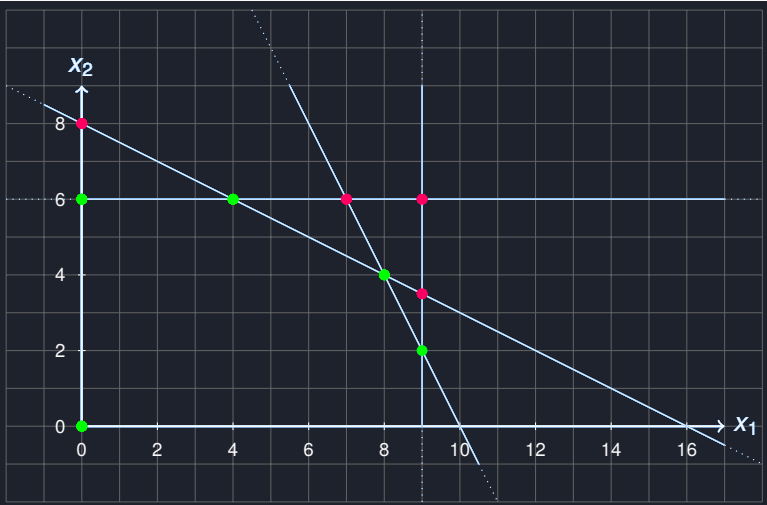
Linear Optimization



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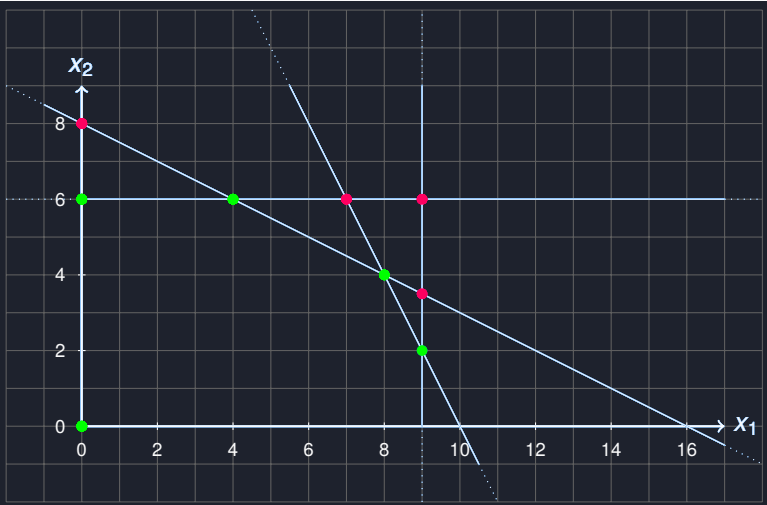
Linear Optimization



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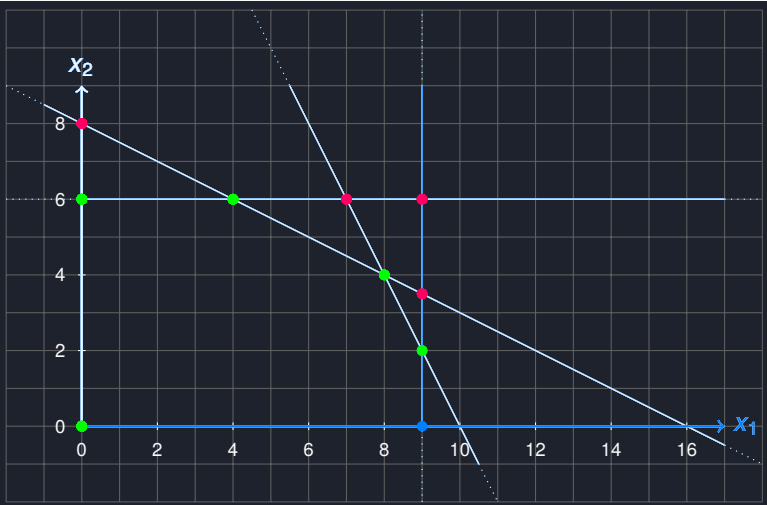
EP Candidate 10:
$c_1 = c_4 \Rightarrow (9, 2)$
TF(9,2)=9+2=11

Linear Optimization



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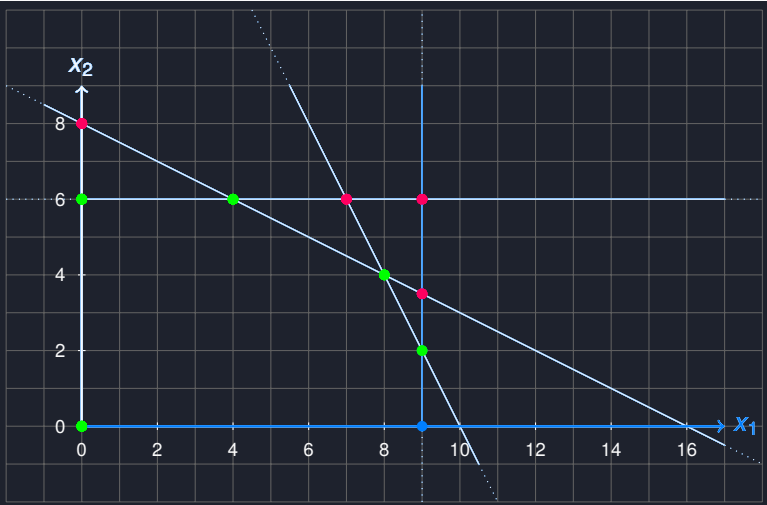
EP Candidate 11:
$x_1 = c_1 \Rightarrow (9, 0)$

Extremal Points: TF(0,0)=0, TF(0,6)=6, TF(4,6)=10, TF(8,4)=12, TF(9,2)=11

Linear Optimization



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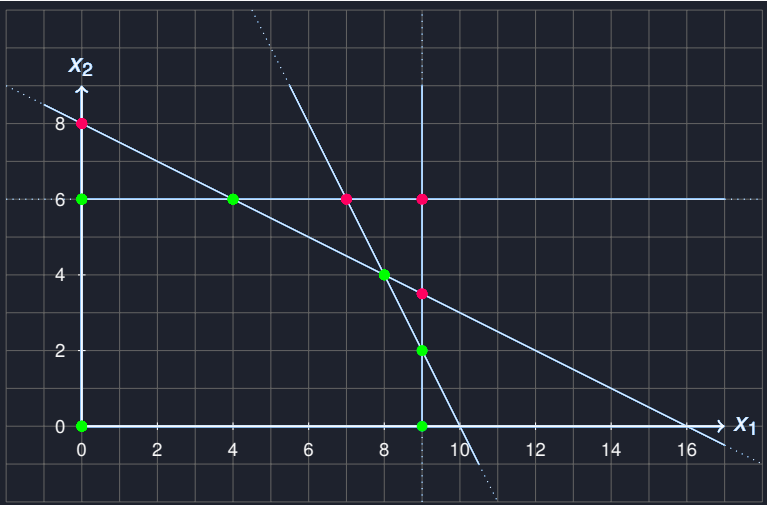
Linear Optimization



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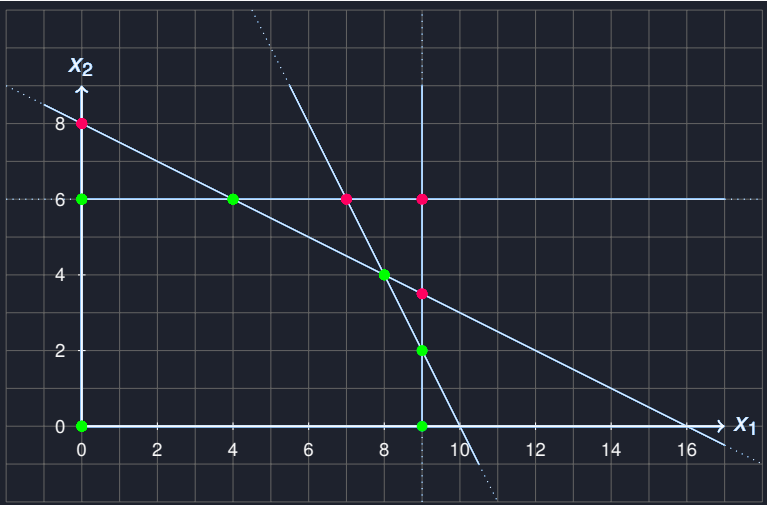
Linear Optimization



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EP Candidate 11:
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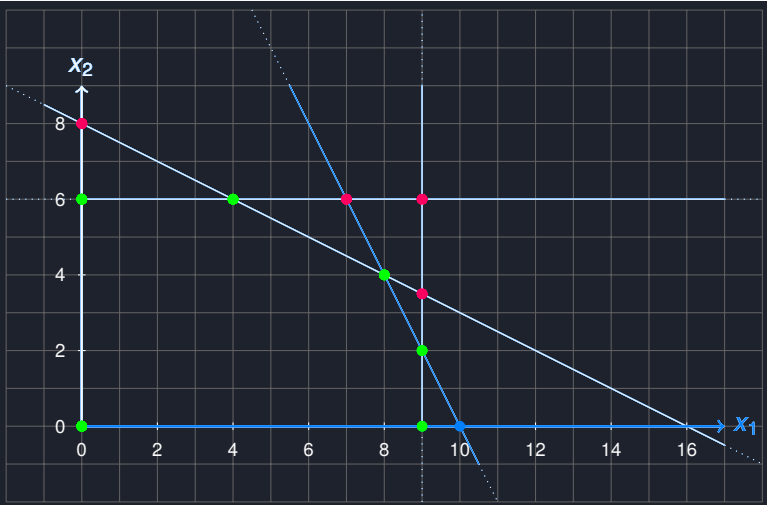
TF(9,0)=9+0=9

Extremal Points: TF(0,0)=0, TF(0,6)=6, TF(4,6)=10, TF(8,4)=12, TF(9,2)=11

Linear Optimization



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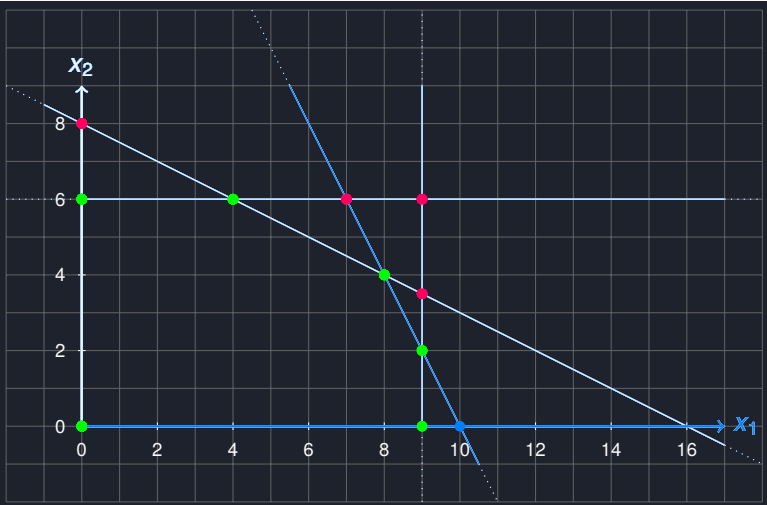
EP Candidate 12:
$x_1 = c_4 \Rightarrow (10, 0)$

Extremal Points: TF(0,0)=0, TF(0,6)=6, TF(4,6)=10, TF(8,4)=12, TF(9,2)=11, TF(9,0)=9

Linear Optimization



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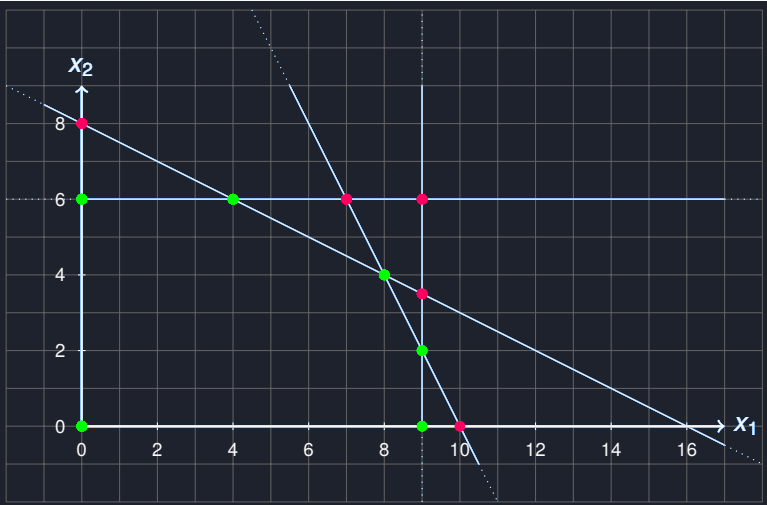
Linear Optimization



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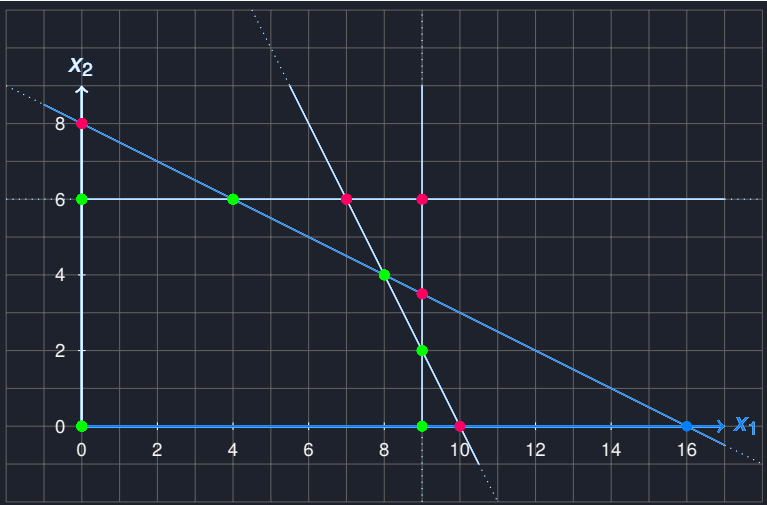

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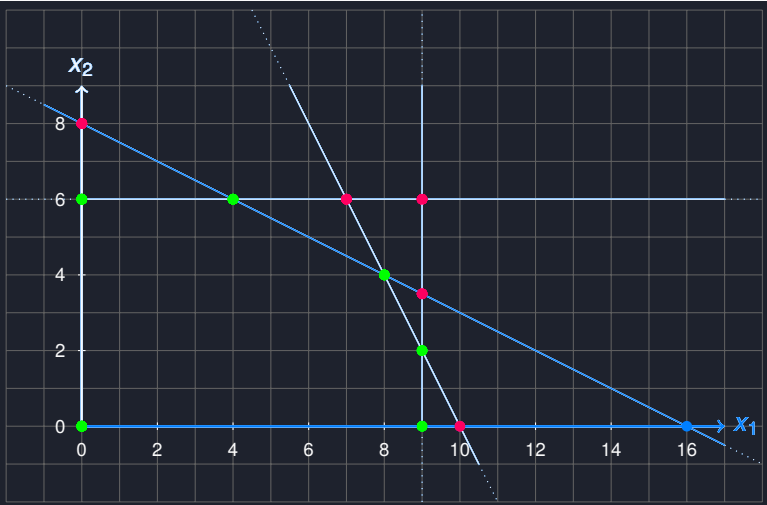
EP Candidate 13:
$x_1 = c_3 \Rightarrow (16, 0)$

Extremal Points: TF(0,0)=0, TF(0,6)=6, TF(4,6)=10, TF(8,4)=12, TF(9,2)=11, TF(9,0)=9

Linear Optimization



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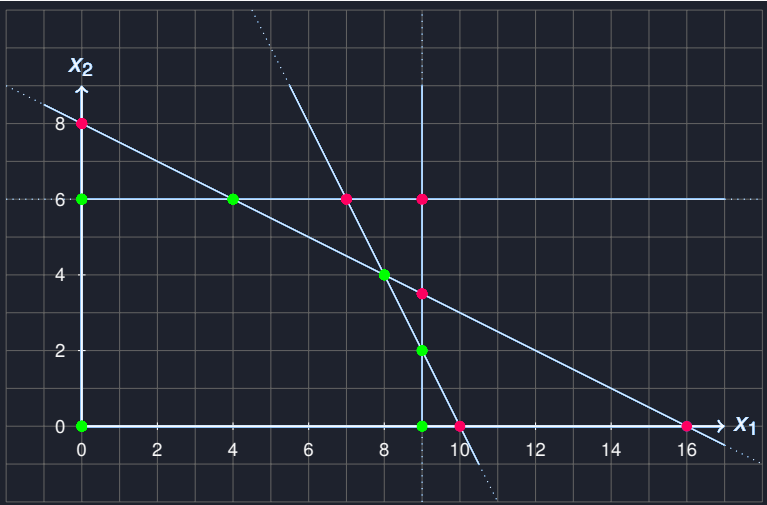
Linear Optimization



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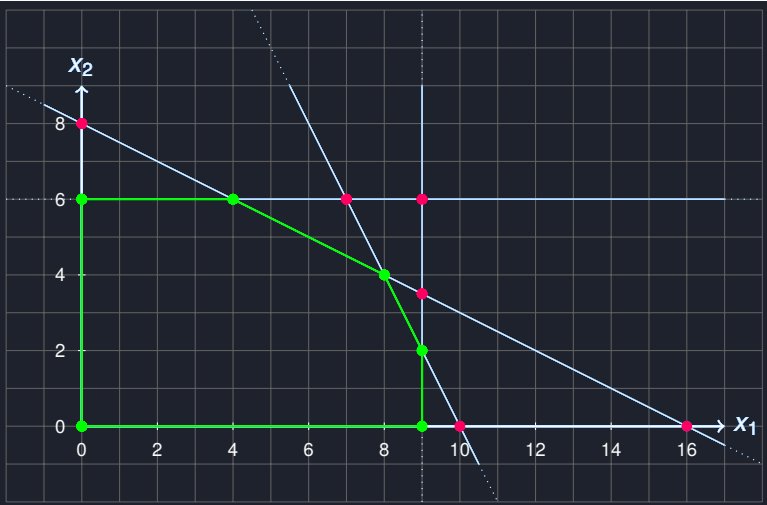

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That's all Folks!