INFORMATION FOR THE COURSE MAC 569 "CONVEX OPTIMIZATION - THEORY AND ALGORITHMS"

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1. General Outline

Optimization problems arise in any kind of practical problem in the sciences, engineering and economics. Nowadays, powerful numerical solvers are available to the end-user and therefore optimization tools are widely available. At first sight it seems that there is thus no need to investigate optimization problems in detail. Unfortunately, it turns out that most optimization problems in practice are *unsolvable*. Therefore, the creation of solvable optimization models is a key issue in practice. One large and widely-used class of "solvable" optimization problems are *convex optimization* problems. In this lecture we give a first introduction to this important class of optimization problems, and study numerical methods that have been developed for solving convex optimization efficiently in large-scale applications.

2. Topics

The following topics shall be discussed in this course

- Elements of Convex Analysis
- Examples of Convex Optimization problems.
- Duality
- Methods for Unconstrained Minimization
- A primer on Monotone Operator Theory
- Primal-Dual Splitting Methods

3. Structure of the Course

The first part of the course is intended to be an introduction to convex analysis, at the level needed to comprehend the notions and concepts introduced in this course. We then move on and give a detailed account on the fundamental notion of duality in optimization. From there on, we embark our study of numerical methods for convex optimization problems. Starting with classical concepts for unconstrained optimization (Gradient descent, Newton Method), we directly

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move one to work out a unified mathematical framework for designing and analyzing algorithms in the convex world. This general framework uses the language developed for (maximally) *monotone operators*. The advantage of this formalism is that we learn a versatile concept that is not only suitable for optimization purposes, but gives us a general apparatus for equilibrium problems and variational inequalities as they arise as abstract formulations in non-linear functional analysis. We stick here to finite-dimensions to make the material accessible.

This course consists of lectures and reading classes. In the first lectures we will introduce the fundamentals of non-smooth and convex analysis. The second part focusses on families of numerical algorithms. In that part of the course, the students are asked to prepare lectures for the audience.

4. Course Dates

There are two meetings planned every week

- Monday, 8:30-10:00, D.002
- Wednesday 12:00-13:30, D.002

5. Grading

The grade for this course consists of two parts:

(1) Successful completion of a student project (50 % of the grade)
(2) Oral Exam at the end of the semester (50 % of the grade)

Topics for student projects will be made available at the beginning of the semester.

6. Literature

This lecture will be accompanied by lecture notes. We follow closely the following excellent monographs:

- Stephen Boyd and Lieven Vandenberghe. Convex Optimization. Cambridge University Press, 2004
- Amir Beck: First-Order Methods in Optimization. MOS-SIAM Series on Optimization.
- Ernest K. Ryu and Wotao Yin. Large-Scale Convex Optimization - Algorithms & Analyses via Monotone Operator Theory. Cambridge University Press, 2023