4/a) $u(x,t) = F(\alpha \cdot x - \mu t)$

$$\partial_{x_j} u = F'(\alpha \cdot x - \mu t)\, \partial_{x_j}(\alpha_1 x_1 + \dots + \alpha_n x_n - \mu t)$$
$$= F'(\dots)\, \alpha_j$$
$$\partial_{x_j}^2 u = \alpha_j^2 F''$$
$$\partial_t^2 u = (-\mu)^2 F''$$

$$\left(\mu^2 - \sum c_j^2 \alpha_j\right) F'' = 0$$

b) $u(x,t) = F(\alpha \cdot x - \mu t) = F(\alpha \cdot x - \mu(\alpha \cdot \alpha) t)$

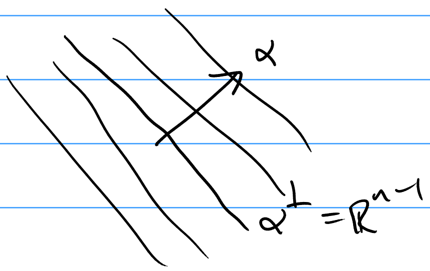
$$= F(\alpha \cdot (x - \mu t \alpha) - \mu 0)$$

$$= u(x - \mu t \alpha, 0)$$

Imagine $\alpha = e_1$

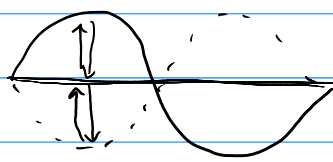
$u(x,t) = F(x_1 - \mu t)$ ← transport solution with speed μ.

independent of $x_2, \dots, x_n$



Plane wave

$\underline{\sin x}\ \underline{\sin t}$ on $[0,\pi]$



<u>Special waves.</u>

~~Spherical waves.~~ $\cancel{u(x,t) = F(|x|)G(t)}$

Standing waves. $u(x,t) = F(x)\sin(\omega t)$

$$-\omega^2 F(x) \sin \omega t - \Delta F(x) \sin \omega t = 0$$

$$\Delta F = -\omega^2 F \qquad \text{for constant frequency } \omega$$

Same idea as taking Fourier transform. $\hat{u}(x,\omega) = \int_{-\infty}^{\infty} u(x,t) e^{-i\omega t}\, dt$

$$(-i\omega)^2 \hat{u} - \Delta \hat{u} = 0 \Rightarrow \Delta \hat{u} = -\omega^2 \hat{u}$$

If $u(x,t)$ only depends on $|x|$

$$r^{N-1}\Delta f = r^{N-1}\frac{\partial^2 f}{\partial r^2} + (N-1)r^{N-2}\frac{\partial f}{\partial r} = \frac{\partial}{\partial r}\left(r^{N-1}\frac{\partial f}{\partial r}\right)$$

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial r^2} - \frac{N-1}{r}\frac{\partial u}{\partial r} = 0$$

$$\cancel{\frac{\partial ru}{\partial r} = \frac{\partial u}{\partial r} + r\frac{\partial u}{\partial r}}$$

$$\frac{\partial^2 ru}{\partial t^2} - C\frac{\partial^2 (ru)}{\partial r^2} = 0$$

42 $\nabla \cdot E = 0$ $\quad \nabla \cdot B = 0$ $\quad \nabla \times E = -\partial_t B$ $\quad \nabla \times B = \varepsilon\mu\, \partial_t E$

$$\nabla \times \nabla \times E = -\partial_t (\nabla \times B) \qquad \nabla \times \nabla \times B = \varepsilon_0\mu_0 \partial_t \nabla \times E$$

$$\nabla(\cancel{\nabla \cdot E}) - \Delta E = -\partial_t(\varepsilon\mu\, \partial_t E)$$

$$\Delta E = \varepsilon\mu\, \partial_t^2 E$$

$\Delta = \nabla \cdot (\nabla\ )$ scalar

$$\partial_t^2 E - \frac{1}{\varepsilon\mu} \Delta E = 0$$

$$\text{speed}^2 = \sum \alpha_j^2 \frac{1}{\sqrt{\varepsilon_0\mu_0}}^2 = \frac{1}{\varepsilon_0\mu_0} \cancel{\sum \alpha_j^2}$$

$$\text{Speed} = 1/\sqrt{\varepsilon_0\mu_0} = 299{,}000 \text{ km/s}$$

Each component is a wave equation $\quad u : \mathbb{R}^n \times \mathbb{R} \to \underline{\mathbb{R}}$
scalar

$$\partial_t^2 E_j - \frac{1}{c^2} \Delta E_j = 0.$$

→ polarization $\quad$ simple sol^n / plane wave sol^n

g and h have compact support.

E is finite, dominated for compact time intervals

43.

$$2E(t) = \int_{-\infty}^{\infty} (\partial_t u)^2 \, dx + \int_{-\infty}^{\infty} (\partial_x u)^2 \, dx$$

$$\cancel{2}E'(t) = \int_{-\infty}^{\infty} \cancel{2}(\partial_t u)(\partial_{tt} u) + \cancel{2}(\partial_x u)(\partial_{xt} u) \, dx$$

$$= \int_{-\infty}^{\infty} (\partial_t u)(\partial_{xx} u) + \partial_x u \, \partial_{xt} u \quad dx$$

$$= \int_{-\infty}^{\infty} \partial_x (\partial_t u \, \partial_x u) \, dx$$

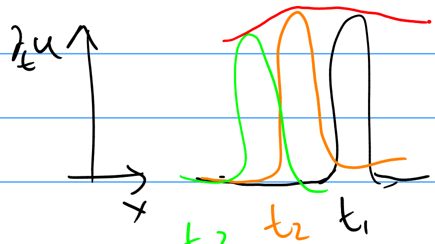
$$= \left[\partial_t u \, \partial_x u \right]_{-\infty}^{\infty}$$

$$= 0.$$

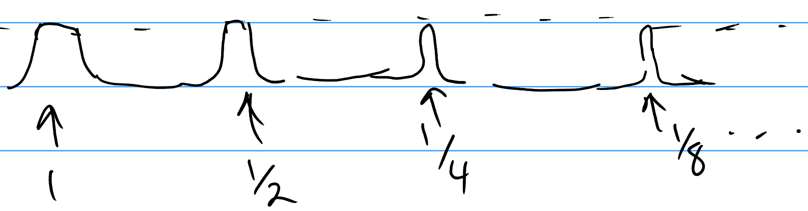
$$E'(t) = \partial_t \int_{-\infty}^{\infty} (\partial_t u)^2 + (\partial_x u)^2 \, dx$$

$\exists g(x)$ $\quad (\partial_t u)^2 + (\partial_x u)^2 \le g(x)$ for $t \in [t_0, t_1]$

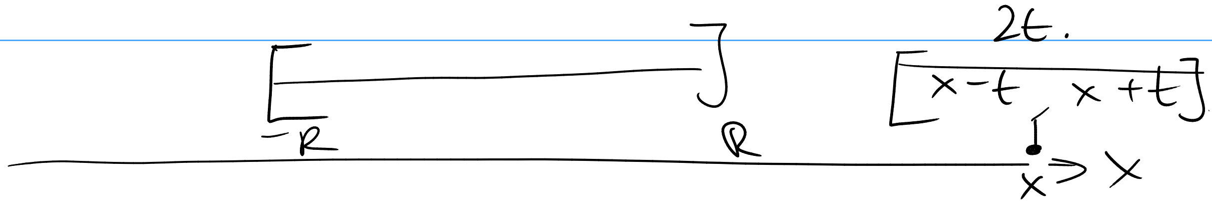
g is L^1



small times large spikes.



$$\partial_{x_1} F = 0 \Rightarrow F(\varphi) = c\left(\int_{-\infty}^{\infty} \varphi(x_1, \ldots, x_n) \, dx_1 \right)$$



If g, h have compact support, then u does, so our proof in part (a) E is constant.

modified (b):

Suppose u_1, u_2 are solutions. $v := u_2 - u_1$

v is a wave.

initial cond v are zero.

$$E(0) = \frac{1}{2}\int_{-\infty}^{\infty} h^2 + \left(\frac{\partial}{\partial x} g\right)^2 dx = 0$$

$$E(t) = \frac{1}{2}\int_{-\infty}^{\infty} (\partial_t v)^2 + (\partial_x v)^2 \, dx = 0.$$

$$\partial_t v = \partial_x v = 0$$

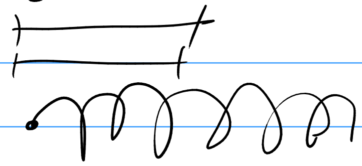
v is constant. $v(x,t) = v_0$

$$0 = v(x,0) = v_0$$

$$v = 0$$

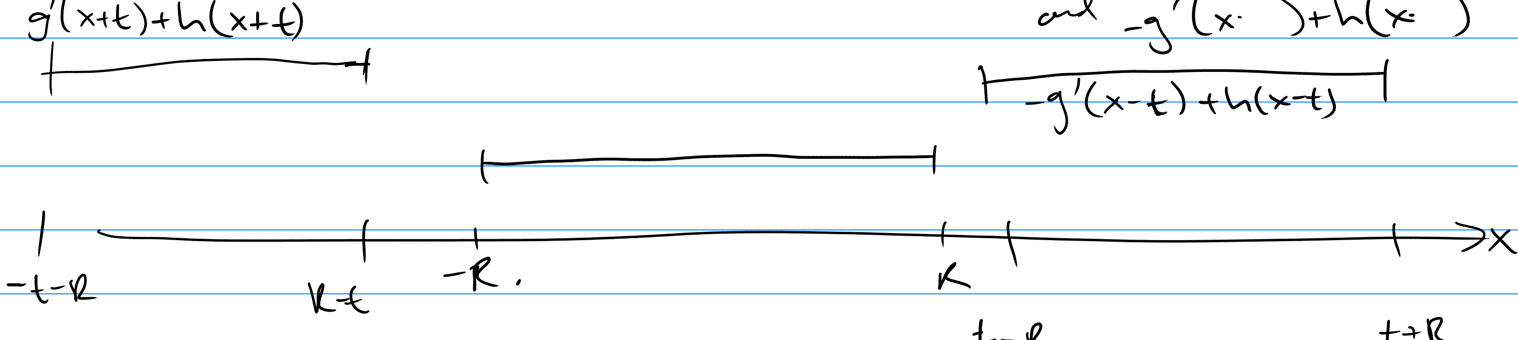
$$u_1 = u_2$$

Spring.

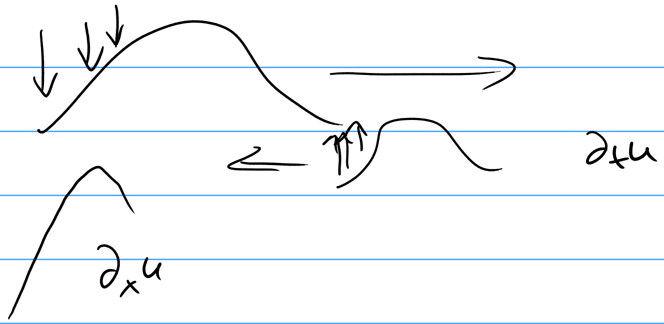


$x'' = -kx \qquad E = \frac{1}{2}kx^2$

g, h are supported on $[-R, R]$ so is $g'(x\ ) + h(x\ )$ and $-g'(x\ ) + h(x\ )$



$t > R.$


$\partial_t u$
$\partial_x u$