$$f''(u_0(x))\, u_0'(x) > -\alpha$$

then exists sol^n $[0,\alpha^{-1})$

$$\dot u + \overset{f'(u)}{u}\, \partial_x u = 0 \qquad f(u) = \tfrac{1}{2}u^2$$

$$f''(u) = 1$$

$u_0(x) = x$

$u_0'(x) = 1$

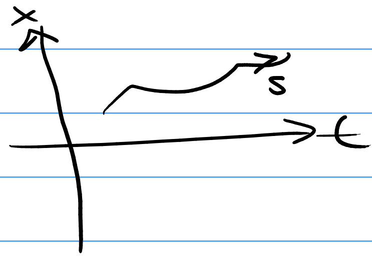
LHS $= 1 \times 1 = 1 > -\alpha$ for any $\alpha > 0$.

a sol^n exists on any interval $[0,\alpha^{-1})$

$$\bigcup_{\alpha>0} [0,\alpha^{-1}) = [0,\infty) \quad \text{when "}\alpha = 0\text{"}$$

f,b) Method of characteristics

$(x(s), t(s))$



Suppose $z(s) = u(x(s), t(s))$

$$\frac{d}{ds} z = \partial_x u \frac{dx}{ds} + \partial_t u \frac{dt}{ds} = 0$$

$$\partial_x u \cdot u + \partial_t u \cdot 1 = 0$$

$$= \partial_x u\, \dot x + (-\partial_x u \cdot u)\cdot \dot t$$

$$= (\partial_x u)(\dot x - u \dot t) = 0$$

Need $\dot x - u\dot t = 0$ necessary.

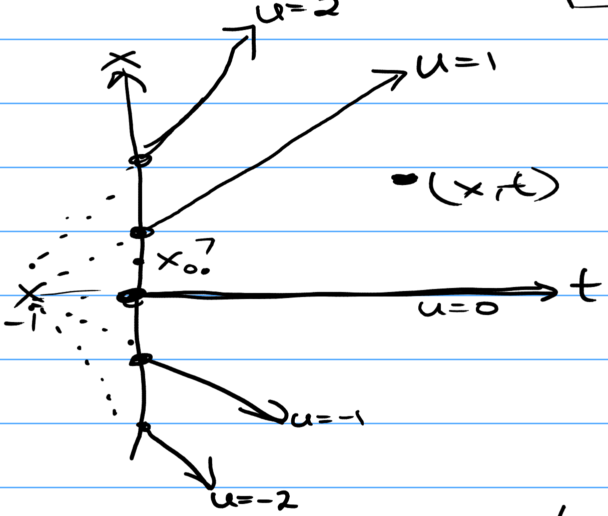
$$\frac{dx}{ds} = u \qquad \frac{dt}{ds} = 1 \qquad \text{choice}$$

$t = s$ ~~x~~ $\qquad \frac{dx}{dt} = u = u(\overbrace{x(0)}^{x_0}, 0) = u_0(x_0) = x_0$

$$\boxed{x = x_0 t + x_0} = x_0(t+1)$$

$$x_0 = \frac{x}{1+t}$$

$t > 0$



$$u(x(t), t) = u(x(0), 0) = x_0$$

$$u(x,t) = x_0$$

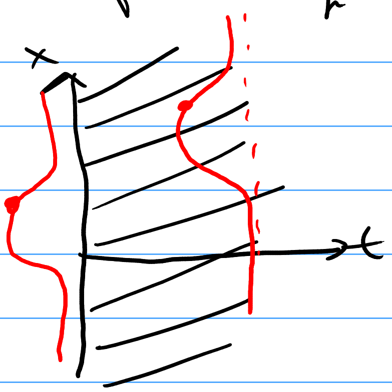
$$= \frac{x}{t+1}$$

e) "well-defined" = no singularities, function exists.

$\ln(-1) \qquad \frac{1}{0}$

$t > 0 \qquad \frac{x}{t+1}$ no singularities.

Transport equation $\quad \dot{u} + b\,\partial_x u = 0$



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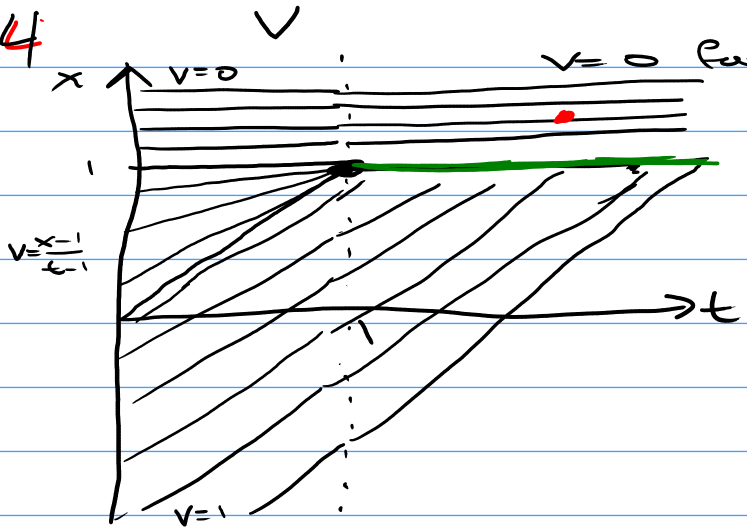
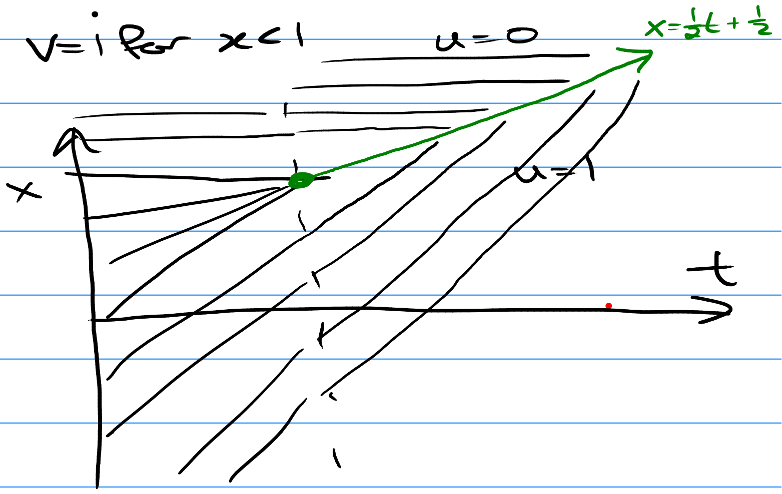




Interpretation is that u is a density of substance. u is air density

$$\int_a^b u = \text{total amount of air/gas in } [a,b]$$

$$v^r = 0 \qquad v^l = 1 \qquad f(u) = \tfrac{1}{2}u^2$$

R-H condition
$$\frac{f(v^r) - f(v^l)}{v^r - v^l} = \frac{\frac{1}{2}0^2 - \frac{1}{2}1^2}{0-1} = \frac{1}{2}$$

$$y(t) = 1 \quad \text{for } t > 1$$

$$\dot{y} = 0$$ R-H condition is <u>not</u> satisfied

We know there <u>is</u> a solution which obeys R-H condition.

Difference total amount of stuff at time t

$$= \int_{-\infty}^{\infty} v-u \, dx = \int_{-\infty}^{1} + \int_{1}^{1+\frac{t-1}{2}} \int_{1+\frac{t-1}{2}}^{\infty} v-u \, dx$$

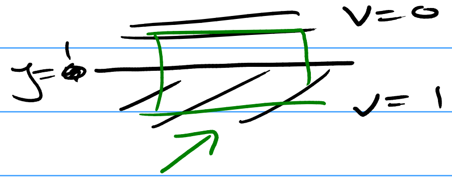
$$= \int_{-\infty}^{1} 1 - 1 \, dx + \int_{1}^{1+\frac{t-1}{2}} 0-1 \, dx + \int_{1+\frac{t-1}{2}}^{\infty} 0-0 \, dx$$

$$= 0 + -x \Big|_{1}^{1+\frac{t-1}{2}} + 0$$

$$= -\frac{t-1}{2} = -\frac{1}{2}t + \frac{1}{2}$$ $\frac{t-1}{2}$ less stuff

<u>alternate way</u>

$$\text{stuff} = \int_0^2 v = \int_0^1 1 + \int_1^2 0 = 1$$



$$\text{flux} = f(u(0,t)) - f(u(2,t))$$
$$= \frac{1}{2} 1^2 - \frac{1}{2} 0^2 = \frac{1}{2}$$

$\boxed{\text{always } 1}$

↑ $\frac{1}{2}$ per second

$\frac{1}{2}$ unit air /sec is disappearing flux = speed × density

S, u

$u = u_m \qquad S = 0$

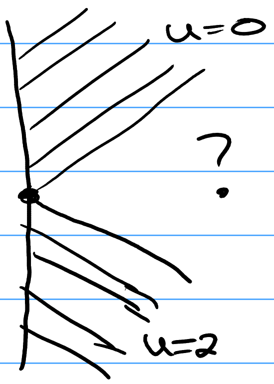
$u = 0 \qquad S = S_m$

$$S = S_m\left(1 - \frac{u}{u_m}\right)$$

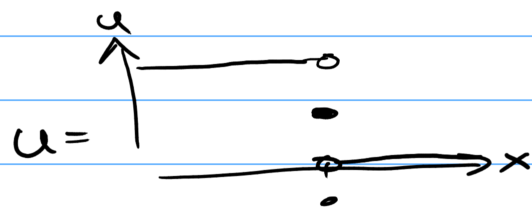
$$\dot{u} + f'(u)\,\partial_x u = 0$$

$$\dot{u} + S_m\left(1 - 2\frac{u}{u_m}\right)\partial_x u = 0$$ let $S_m = 1$ $\quad u_m = 2$

$$\dot{u} + (1-u)\,\partial_x u = 0 \qquad u_0(x) = \begin{cases} 0 & x > 0 \\ 2 & x < 0 \end{cases}$$



$$f = S_m u\left(1 - \frac{u}{u_m}\right) = u\left(1 - \frac{u}{2}\right)$$

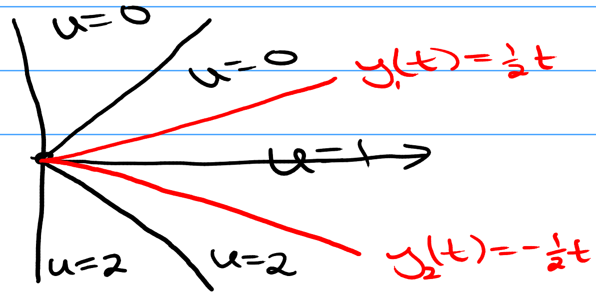


Discontinuous sol^{n}

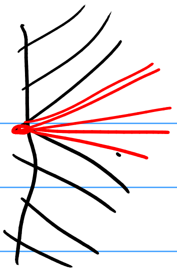
$$u = \begin{Bmatrix} 0 \\ 1 \\ 2 \end{Bmatrix}$$

$$\dot{y}_1 = \frac{1(1-\frac{1}{2}) - 0(1-0)}{1-0} = \frac{1}{2}$$

$$\dot{y}_2 = \frac{2(1-\frac{2}{2}) - 1(1-\frac{1}{2})}{2-1} = -\frac{1}{2}$$



"rarefied"



"new" characteristics $x = ct$ for $c \in (-1, 1)$

$$u(x,t) = g(c) = g\left(\frac{x}{t}\right) \qquad g(-1) = 2, \quad g(1) = 0$$

$$\partial_t u + (1-u)\partial_x u = 0$$

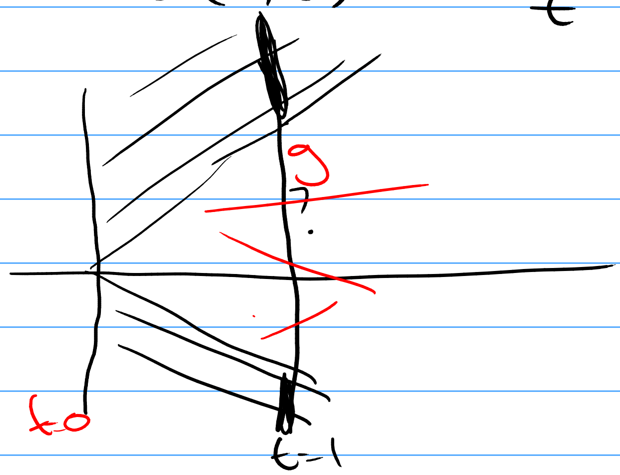
$$-\frac{x}{t^2} g'\left(\frac{x}{t}\right) + (1-g)\left(\frac{1}{t}\right) g'\left(\frac{x}{t}\right) = 0$$

$$\frac{1}{t} g' \left[-\frac{x}{t} + (1-g) \right] = 0$$

$$g' \times [-c + 1 - g] = 0$$

$$g = 1 - c$$

$$u(x,t) = 1 - \frac{x}{t}$$



$$\dot{u} + (1-u)\partial_x u = 0$$

$$\dot{x} = \frac{dx}{ds}$$

$$\frac{d}{ds} z = \partial_t z \, \dot{t} + (\partial_x z) \dot{x}$$

$\dot{t} = 1$ $\qquad \dot{x} = 1 - u$ $\qquad \dot{z} = 0$ $\qquad z = u(x(s), t(s))$

$t = s$ $\qquad = 1 - u_0(x_0)$

$$x = (1 - u_0(x_0)) t + x_0$$

$$u_0 = \begin{cases} 0 \\ 2 \end{cases}$$