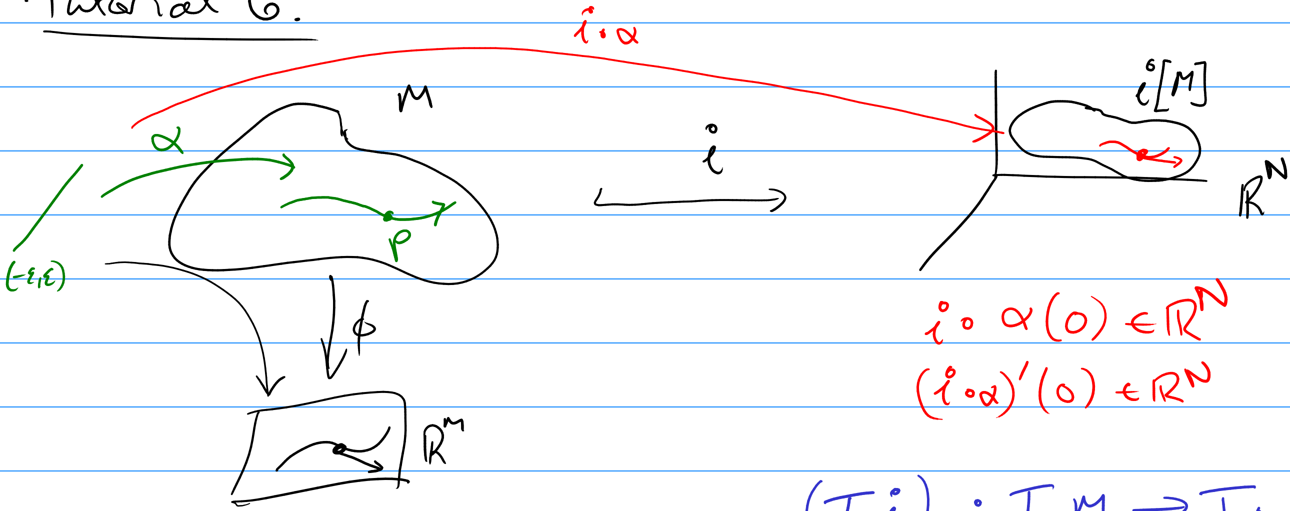
Tutorial 6.



$i \circ \alpha(0) \in \mathbb{R}^N$

$(i \circ \alpha)'(0) \in \mathbb{R}^N$

$(T_p i) : T_p M \longrightarrow T_{i(p)} \mathbb{R}^N = \mathbb{R}^N$

$(T_p i)([\alpha]) = [i \circ \alpha]$

$= (i \circ \alpha)'(0).$

$\alpha(0) = p$

Tangent vector $v = [\alpha] \in T_p M$

$$i : S^n \longrightarrow \mathbb{R}^{n+1} \qquad x \in S^n$$

$$T_x i : T_x S^n \longrightarrow T_x \mathbb{R}^{n+1} = \mathbb{R}^{n+1}$$

Choose a vector $[\alpha] \in T_x S^n$

$\alpha(t) \in S^n$

i.e. $\alpha(t) \in \mathbb{R}^{n+1}$ so that

$\|\alpha(t)\| = 1$

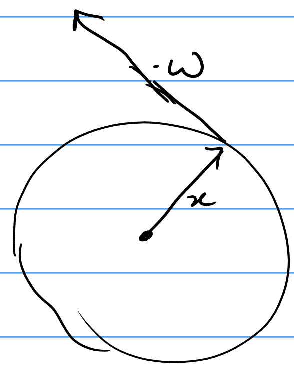
$$T_x i([\alpha]) = [i \circ \alpha] = (i \circ \alpha)'(0)$$

$$\alpha(t) \cdot \alpha(t) = 1$$

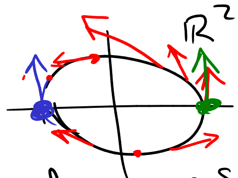
$$\alpha'(t) \cdot \alpha(t) + \alpha(t) \cdot \alpha'(t) = 0$$

$$\alpha(t) \cdot \alpha'(t) = 0$$

$$\underbrace{\alpha(0)}_{x} \cdot \underbrace{\alpha'(0)}_{w} = 0$$



$$x \cdot w = 0$$



b) The tangent vectors of S^1 at $x \in S^1$ are the vectors w of $\mathbb{R}^2$ with $x \cdot w = 0$.

For each x we choose a $\underline{\underline{w(x)}} \in \mathbb{R}^2$ with $w(x) \cdot x = 0$.

first suggestion
$w : S^1 \to \mathbb{R}^2$ with $w(x) \cdot x = 0$.

second suggest $w : S^1 \longrightarrow \mathbb{R}^2 \times S^1$

$x \longmapsto (\tilde{w}(x), x)$ with $\tilde{w}(x) \cdot x = 0$

basically the definition of a section of a vector bundle.

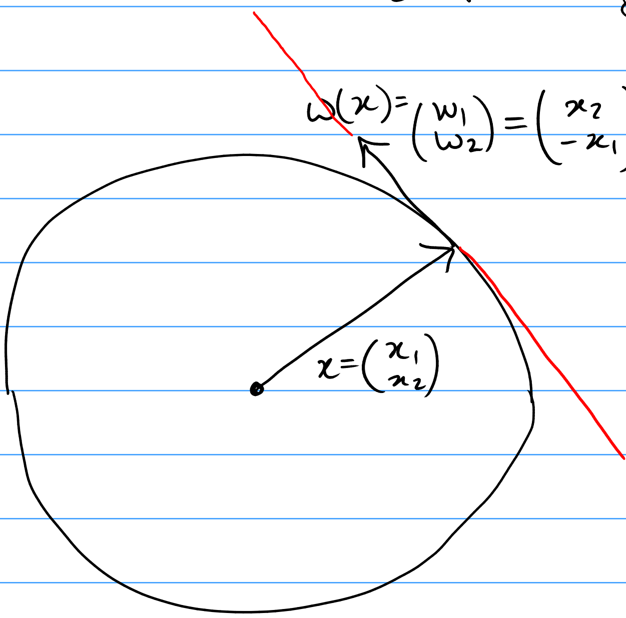
a section is a function $x \mapsto f(x) \in T_x S^1$

π projection on second component.

Def 2.1 Vector field = section of the tangent bundle.

$w(x) = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -x_1 \end{pmatrix}$ $\quad \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$

$w_1 x_1 + w_2 x_2 = 0$
$\quad x_2 \qquad -x_1$



$\|w(x)\| = \sqrt{(x_2)^2 + (-x_1)^2}$
$= \|x\| = 1.$

$w(x) \neq 0.$

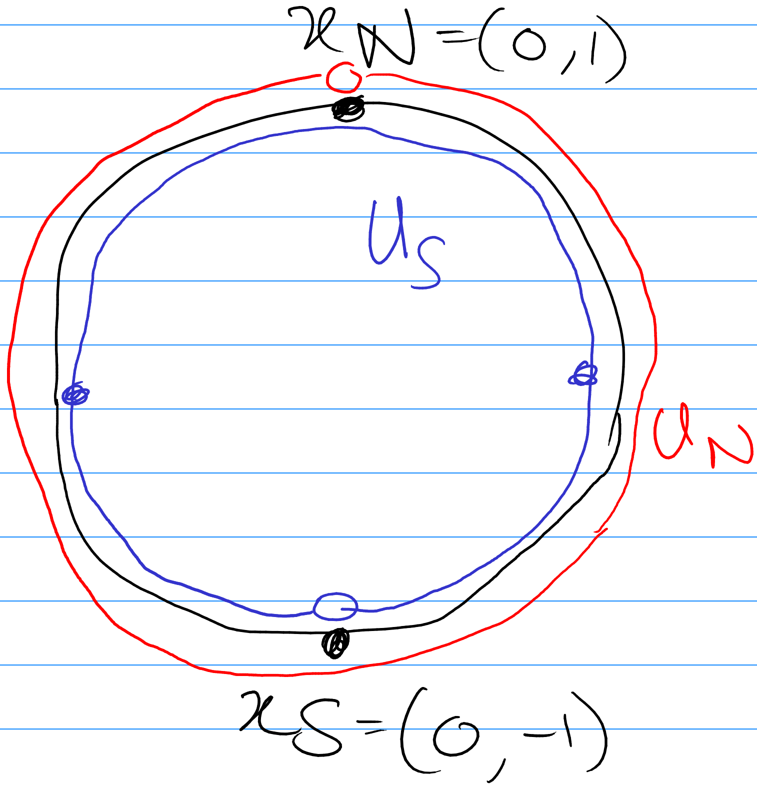
$\{w(x)\}$ is a basis for $T_x S^1$

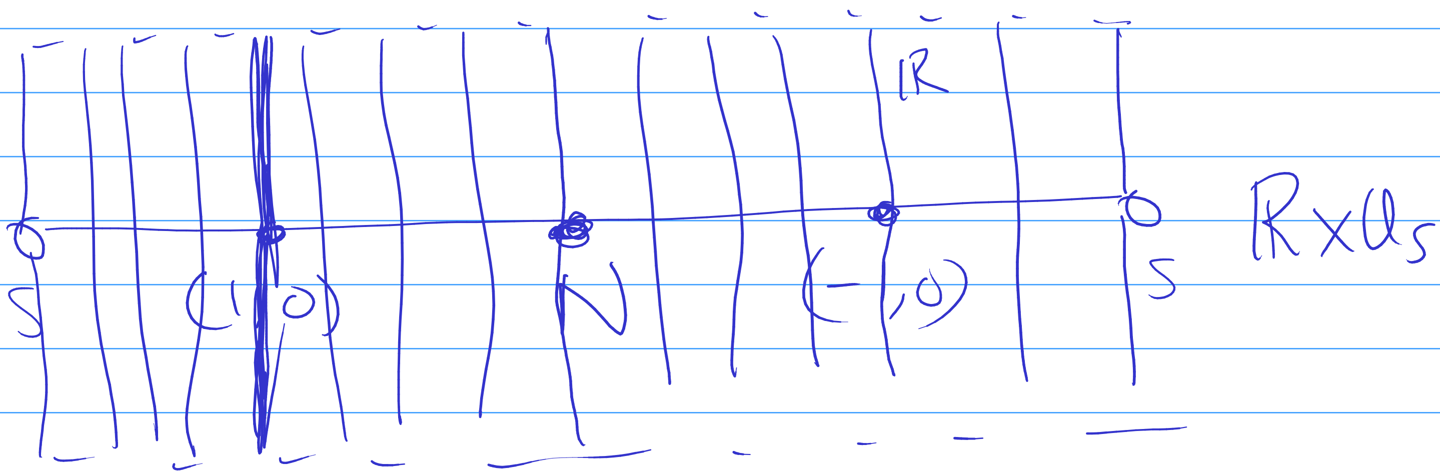
So every vector in $T_x S^1$ is $a \cdot w(x)$ for some $a \in \mathbb{R}$.

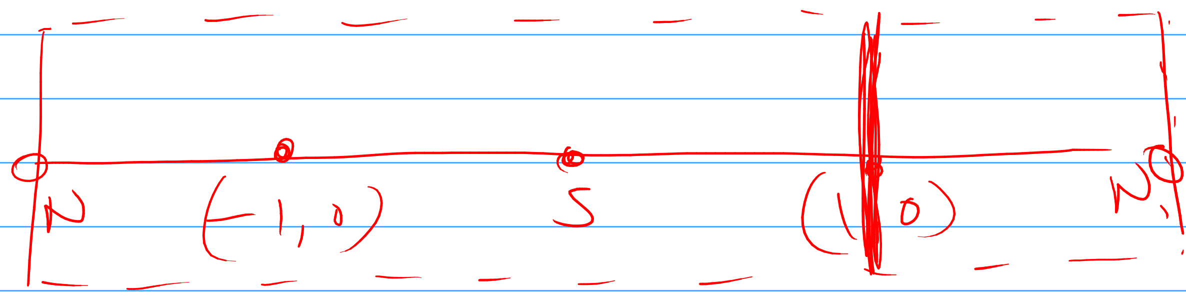
every vector field is $a(x)\, w(x)$ for some real function $a : S^1 \to \mathbb{R}$.

n-dimension
Vector bundle with n linearly independent sections is called trivial.

If we have these n sections then the coefficients are coordinates for the vectors in $T_x S^1 = \mathbb{R} \times S^1$







How does the definition of vector bundle apply to Möbius bundle?

Def 1.49

(X, B, π)

$X = E =$ Möbius band $\qquad X =$ "total space"

$B =$ "base manifold" $= S^1$

$\pi : X \to B$ surjective smooth

it is the map from the fibre to the base point.