



10th Summer school on Lévy Processes

16th July – 17th July 2022

University of Mannheim




Contents

	Program Overview	1
1	Saturday	2
2	Sunday	3
	Titles and Abstracts	4
	Speakers	13



Program Overview

	Saturday	Sunday
9:00-10:30	Lecture: Lévy processes	Lecture: Lévy processes
10:30-10:45	<i>Break</i>	
10:45-11:45	Talk Session 1	Talk Session 4
11:45-12:00	<i>Break</i>	
12:00-13:00	Talk Session 2	Talk Session 5
13:00-14:30	<i>Lunch break</i>	Excursion to Heidelberg
14:30-16:00	Lecture: Lévy processes	
16:00-16:30	<i>Coffee break</i>	
16:30-17:30	Talk Session 3	
17:30-18:00	<i>Break</i>	
18:00-20:00	BBQ	



1. Saturday

Program of the day

Lecture: 9:00–10:30

Rivero, Victor, Centro de Investigacion en Matematicas (CIMAT): *Introduction to Lévy Processes*

Talk Session 1: 10:45–11:45

Bang, D.: *How smooth can a convex hull of a Lévy path be?*

Contreras, J.: *Generalized Scale Functions for Spectrally Negative Lévy Processes*

Gonzalez Cazares, J.: *The local growth of the vertex process of a Lévy Process*

Karbach, S.: *Positive multivariate CARMA processes*

Szulda, G.: *CBI-time changed Lévy processes*

Talk Session 2: 12:00–13:00

Kumar, C.: *Well-posedness and Euler scheme for McKean–Vlasov SDE with super-linear coefficients*

Kumar, T.: *Explicit Milstein-type Scheme for Stochastic Differential Equations with Markovian Switching*

Pasemann, G.: *Statistical analysis of discretely sampled semilinear SPDEs: a power variation approach*

Scheffels, D.: *Stochastic Volterra equations with Hölder diffusion coefficients*

Yuan, S.: *Modulation and amplitude equations on bounded domains for nonlinear SPDEs driven by cylindrical α -stable Lévy processes*

Lecture: 14:30–16:00

Rivero, V.: *Introduction to Lévy Processes*

Talk Session 3: 16:30–17:30

Geiss, S.: *Sharp generalizations of stochastic Gronwall inequalities*

Klump, A.: *The classical and the soft-killing Inverse First-Passage time problem for Brownian motion*

Pedraza Ramirez, J.M.: *Optimal Stopping for Exponential Lévy Models with Weighted Discounting*

Šebek, S.: *Expected volume for the convex hull of the time-space trajectory of Brownian motion*

Valentic, I.: *Skew-product decomposition of Brownian motion on ellipsoid*



2. Sunday

Program of the day

Lecture: 09:00–10:30

Rivero, V.: *Introduction to Lévy Processes*

Talk Session 4: 10:45–11:45

Angtuncio Hernández, O.: *Convergence of multitype Bienaymé-Galton-Watson processes conditioned on the sizes by types*

Buckland, M.: *Branching Interval Partition Diffusions*

Gardini, M.: *The Variance Gamma++ Process and Applications to Energy Markets*

Panzo, H.: *Spectral upper bound for the torsion function of symmetric stable processes*

Santoyo Cano, A.: *A Meyer-Itô formula for stable processes via fractional calculus*

Talk Session 5: 12:00–13:00

Baraniewicz, M.: *Convolutions of radial, exponential densities*

Lazic, P.: *Subgeometric ergodicity of regime-switching diffusion processes*

Śliwiński, M.: *Discrete Feynman–Kac operators for Markov chains with direct step property*

Yilmaz, B.: *Option Pricing in Emerging Markets Using Pure Jump Processes: Explicit Calibration to BIST30 European Index Options*



Titles and Abstracts

When is the convex hull of a Lévy path smooth?

Sat,
Session 1

David Bang

University of Warwick

We characterise, in terms of their transition laws, the class of one-dimensional Lévy processes whose convex hull has a continuously differentiable boundary. We show that this phenomenon is exhibited by a broad class of infinite variation Lévy processes and depends subtly on the behaviour of the Lévy measure at zero. We introduce a class of strongly eroded Lévy processes, whose Dini derivatives vanish at every local minimum of the trajectory for all perturbations with a linear drift, and prove that these are precisely the processes with smooth convex hulls. We study how the smoothness of the convex hull can break and construct examples exhibiting a variety of smooth/non-smooth behaviours. Finally, we conjecture that an infinite variation Lévy process is either strongly eroded or abrupt, a claim implied by Vigon's point-hitting conjecture.

Generalized Scale Functions for Spectrally Negative Lévy Processes

Sat,
Session 1

Jesus Contreras

Centro de Investigacion en Matematicas (CIMAT)

The exit probabilities out of an interval $[b, a]$ are well known for a spectrally negative Lévy process X and are given in terms of scale functions. In this talk, we consider functionals of X and its supremum S up to the exit time T of the interval. It turns out that the Laplace transform of such functionals can be expressed in terms of generalized scale functions and an excursion theory approach will provide an expression for them.

The local growth of the vertex process of a Lévy process

Sat,
Session 1

Jorge Gonzalez Cazares

University of Warwick

The convex minorant of a Lévy process is known to be piecewise linear. The right-inverse of its right-derivative is known as the vertex process. In this talk we will establish sufficient and necessary conditions to determine when a function is an upper or lower function for the vertex process either at time zero or at the minimum of the Lévy process.

Positive multivariate CARMA processes

Sven Karbach
University of Amsterdam

Sat,
Session 1

In this talk we consider multivariate continuous-time autoregressive moving-average (MCARMA) processes with values in convex cones. More specifically, we introduce matrix-valued MCARMA processes with Lévy noise and present necessary and sufficient conditions for processes from this class to be cone valued. We present specific hands-on conditions in the following two cases: First, for classical MCARMA with values in the positive orthant of the d -dimensional Euclidean space. Second, for MCARMA processes on real square matrices taking values in the cone of symmetric and positive semi-definite matrices. In addition to the above, we discuss applications and examples of positive MCARMA processes.

CBI-time-changed Lévy processes

Guillaume Szulda
Università degli studi di Padova

Sat,
Session 1

We introduce and study the class of CBI-time-changed Lévy processes (CBITCL), obtained by time-changing a Lévy process with respect to an integrated continuous-state branching process with immigration (CBI). We characterize CBITCL processes as solutions to a certain stochastic integral equation and relate them to affine stochastic volatility processes. We provide a complete analysis of the time of explosion of exponential moments of CBITCL processes and study their asymptotic behavior. In addition, we show that CBITCL processes are stable with respect to a suitable class of equivalent changes of measure. As illustrated by some examples, CBITCL processes are flexible and tractable processes with a significant potential for applications in finance.

Well-posedness and Euler scheme for McKean–Vlasov SDE with super-linear coefficients

Chaman Kumar
Indian Institute of Technology Roorkee

Sat,
Session 2

We first establish well-posedness of McKean–Vlasov stochastic differential equations (McKean–Vlasov SDEs), possibly with coefficients of super-linear growth in the state variable. Then, we present stable time-stepping explicit tamed Euler for an interacting particle system associated with the McKean–Vlasov equation. We prove stability and strong convergence of order $1/2$.

The talk is based on joint work with Neelima, W. Stockinger, and C. Reisinger.

Explicit Milstein-type Scheme for Stochastic Differential Equation with Markovian Switching

Sat,
Session 2

Tejinder Kumar

Indian Institute of Technology Roorkee

Stochastic differential equation with Markovian switching (SDEwMS) has found several applications in finance, biology, physics and other domains. Often, explicit solutions of such equations are not known and hence it becomes necessary to find their approximate solutions. An explicit Milstein-type scheme for stochastic differential equation with Markovian switching is developed without employing the Itô-Taylor expansion, with minimal regularity restrictions on both coefficients. To be more specific, coefficients are presumed to be once differentiable. Further, its strong convergence in L^2 -sense is established. This technique is helpful to tackle two-fold difficulties arising due to jumps of the Markov chain and the reduction of regularity requirements on the coefficients. This is joint work with Chaman Kumar.

C. Kumar and T. Kumar (2021). A note on explicit Milstein type scheme for stochastic differential equation with Markovian switching, *Journal of Computational and Applied Mathematics*, 395, 113594.

Statistical analysis of discretely sampled semilinear SPDEs: a power variation approach

Sat,
Session 2

Gregor Pasemann

Humboldt University of Berlin

We consider a parameter estimation problem for semilinear stochastic partial differential equations (SPDE) from discrete observations, adopting a power variation approach. The varying spatial smoothness of the solution process requires a non-trivial regularity-dependent adjustment. In order to understand this effect systematically, we study representations of the solution process in terms of iteratively integrated fractional Brownian motion. The central limit theorems obtained for these reference processes are of independent interest and relate to the SPDE setting. The general theory is applicable to a broad class of models, including stochastic reaction-diffusion equations and the stochastic Burgers equation. This talk is based on joint work with Igor Cialenco and Hyun-Jung Kim.

Stochastic Volterra equations with Hölder diffusion coefficients

Sat,
Session 2

David Scheffels

Universität Mannheim

The existence of strong solutions and pathwise uniqueness are established for stochastic Volterra equations (SVEs) with Hölder continuous diffusion coefficients and sufficiently regular kernels. Moreover, we introduce a Volterra local martingale problem for SVEs with fairly general kernels to prove weak existence.

Modulation and amplitude equations on bounded domains for nonlinear SPDEs driven by cylindrical α -stable Lévy processes

Sat,
Session 2

Shenglan Yuan
Universität Augsburg

In the present work, we establish the approximation via modulation or amplitude equations of nonlinear stochastic partial differential equation (SPDE) driven by cylindrical α -stable Lévy processes.

We study SPDEs with a cubic nonlinearity, where the deterministic equation is close to a change of stability of the trivial solution. The natural separation of time scales close to this bifurcation allows us to obtain an amplitude equation describing the essential dynamics of the bifurcating pattern, thus reducing the original infinite dimensional dynamics to a simpler finite-dimensional effective dynamics. In the presence of a multiplicative stable Lévy noise that preserves the constant trivial solution we study the impact of noise on the approximation.

In contrast to Gaussian noise, where non-dominant pattern are uniformly small in time due to averaging effects, large jumps in the Lévy noise might lead to large error terms, and thus new estimates are needed to take this into account.

Sharp generalizations of stochastic Gronwall inequalities

Sat,
Session 3

Sarah Geiss
TU Berlin

We provide nonlinear generalizations of a class of stochastic Gronwall inequalities that has been studied by von Renesse and Scheutzow (2010), Scheutzow (2013), Xie and Zhang (2020) and Mehri and Scheutzow (2021). This class of inequalities is useful to study path-dependent SDEs driven by Lévy processes. As a corollary, we obtain new existence and uniqueness results and a new criterion for the finiteness of exponential moments for such Lévy driven SDEs.

More precisely, we study generalizations of the Bihari-LaSalle type: Let $\{X_t, t \geq 0\}$ be a càdlàg non-negative process, satisfying

$$X_t \leq \int_0^t \eta(X_{s^-}^*) dA_s + M_t + H_t$$

where $X_s^* := \sup_{u \leq s} X_u$ denotes the running supremum, M is a local martingale starting in 0 and $A : [0, \infty) \mapsto [0, \infty)$ is a non-decreasing càdlàg function. Furthermore, let $\eta : [0, \infty) \mapsto [0, \infty)$ be a suitable nondecreasing function and H a suitable stochastic process.

Our aim is to provide upper bounds for $E[\sup_{t \in [0, T]} X_t^p]$ for $p \in (0, 1)$ which do not depend on the local martingale M . We obtain different types of bounds depending on if η is concave or convex.

[1] Sarah Geiss and Michael Scheutzow, Sharpness of Lenglart's domination inequality and a sharp monotone version, *Electron. Commun. Probab.* 26 (2021), Paper No. 44, 8.

[2] Sarah Geiss, Sharp convex generalizations of stochastic Gronwall inequalities, 2022, arXiv:2112.05047v2.

[3] Sarah Geiss, Concave and other generalizations of stochastic Gronwall inequalities, 2022, arXiv:2204.06042.

The classical and the soft-killing Inverse First-Passage time problem for Brownian motion

Alexander Klump
Universität Paderborn

Sat,
Session 3

The first-passage time of a process is the first time the process crosses a given time-dependent boundary. The classical first-passage time problem asks for the distribution of this stopping time. In the inverse problem the distribution is given and the task is to find a time-dependent boundary such that its first-passage time has the given distribution. The classical questions arising from this problem are the analysis of existence and uniqueness of boundary solutions and the properties of such solutions. In this talk we will be concerned with this classical problem in probability and its soft-killing variant and discuss results obtained by a new stochastic order approach.

Optimal Stopping for Exponential Lévy Models with Weighted Discounting

Jose Manuel Pedraza Ramirez
University of Waterloo

Sat,
Session 3

We consider in this talk an optimal stopping problem with weighted discounting, and the state process is modelled by a general exponential Lévy process. Due to the time inconsistency, we provide a new martingale method based verification theorem for the equilibrium stopping strategies. As an application, we generalize an investment problem with non-exponential discounting studied by Grenadier and Wang (2007) and Ebert et al. (2020) to Lévy models. Closed-form equilibrium stopping strategies are derived, which are closely related to the running maximum of the state process. The impacts of discounting preferences on the equilibrium stopping strategies are examined analytically.

Expected volume of the convex hull of the time-space trajectory of Brownian motion

Stjepan Šebek
University of Zagreb

Sat,
Session 3

In this talk, we will develop a closed formula for the expected value of the d -dimensional volume of the convex hull spanned by the time-space trajectory of $(d - 1)$ -dimensional Brownian motion run up to time one.

Skew-product decomposition of Brownian motion on ellipsoid

Ivana Valentic
University of Zagreb

Sat,
Session 3

We obtain a skew-product decomposition of a Brownian motion on an ellipsoid of dimension n in a Euclidean space of dimension $n + 1$. We prove that the projection of this Brownian motion on to the last coordinate is, after a suitable transformation, a Wright-Fisher diffusion process with atypical selection coefficient.

Convergence of multitype Bienaymé-Galton-Watson processes conditioned on the sizes by types

Sun,
Session 4

Oswaldo Angtuncio Hernández
University Duisburg-Essen

In this paper we consider a multitype Bienaymé-Galton-Watson (MGW) forest, with d types, conditioned on having n_i vertices of type i for each of its types $i \in [d]$. This model is a natural generalization of unitype Bienaymé-Galton-Watson trees conditioned to have size n , which were shown by Aldous to converge to the Continuum Random Tree (1991), as n goes to infinity. First, under some general conditions, we obtain the law after rescaling of (O_1, \dots, O_d) , where O_i is the total number of individuals of type i in the MGW forest. This generalizes the Otter-Dwass (1969) and Kemperman's formula (1950). Then, we prove that the breadth-first walk of the forest, which encodes the whole genealogy, converges to a multidimensional first-passage bridge, generalizing a result of Chaumont-Pardo (2009). Finally, we prove that the profile (the number of individuals by types at each generation) converges, which generalizes a result by Caballero-Pérez-Urbe Bravo (2017). Our results can be viewed as characteristics of a (not yet defined) object, that we call *multitype Lévy forest*.

Branching Interval Partition Diffusions

Sun,
Session 4

Matthew Buckland
University of Oxford

We construct an interval-partition-valued diffusion from a collection of excursions sampled from the excursion measure of a real-valued diffusion, and we use a spectrally positive Lévy process to order both these excursions and their start times. At any point in time, the interval partition generated is the concatenation of intervals where each excursion alive at that point contributes an interval of size given by its value. Previous work by Forman, Pal, Rizzolo and Winkel considers self-similar interval partition diffusions - and the key aim of this work is to generalise these results by dropping the self-similarity condition. The interval partition can be interpreted as an ordered collection of individuals (intervals) alive that have varying characteristics and generate new intervals during their finite lifetimes, and hence can be viewed as a class of Crump-Mode-Jagers-type processes.

The Variance Gamma++ Process and Applications to Energy Markets

Sun,
Session 4

Matteo Gardini

Università di Genova

The purpose of this article is to introduce a new Lévy process, termed the Variance Gamma++ process, to model the dynamics of assets in illiquid markets. Such a process has the mathematical tractability of the Variance Gamma process and is obtained by applying the self-decomposability of the gamma law. Compared to the Variance Gamma model, it has an additional parameter representing the measure of the trading activity. We give a full characterization of the Variance Gamma++ process in terms of its characteristic triplet, characteristic function and transition density. In addition, we provide efficient path simulation algorithms, both forward and backward in time. We also obtain an efficient "integral-free" explicit pricing formula for European options. These results are instrumental to apply Fourier-based option pricing and maximum likelihood techniques for the parameter estimation. Finally, we apply our model to illiquid markets, namely to the calibration of European power futures market data. We accordingly evaluate exotic derivatives using the Monte Carlo method and compare these values to those obtained using the Variance Gamma process and give an economic interpretation of the obtained results. Finally, we illustrate an extension to the multivariate framework.

Spectral upper bound for the torsion function of symmetric stable processes

Sun,
Session 4

Hugo Panzo

Technion – Israel Institute of Technology

Bounds on the product of the principal eigenvalue of the Dirichlet Laplacian and the supremum of the torsion function of Brownian motion that are uniform over a given class of Euclidean domains have been a topic of active research with several improvements and conjectures appearing in the literature recently. We make some progress in the case of the Dirichlet fractional Laplacian and symmetric stable process by deriving an analogous bound that improves upon the existing result and which captures the correct order of growth in the dimension.

A Meyer-Itô formula for stable processes via fractional calculus

Sun,
Session 4

Alejandro Santoyo Cano

IMATE, UNAM

The infinitesimal generator of a strictly α -stable process can be represented as a weighted sum of Riemann-Liouville fractional derivatives of order α and one obtains the fractional Laplacian in the symmetric case. Using this relationship, we compute the inverse of the infinitesimal generator on an adequate function space, from which we can recover the potential if $\alpha \in (0, 1)$ and the recurrent potential if $\alpha \in (1, 2)$. The inverse of the infinitesimal generator is expressed in terms of a linear combination of Riemann-Liouville fractional integrals of order α . One can then state a class of functions that give semimartingales when applied to strictly stable processes and we state a Meyer-Itô theorem with a non-zero (occupational) local time term, providing a generalization of the Tanaka formula given by Tsukada. This result is used to find a Doob-Meyer or semimartingale decomposition for $|X_t - x|^\gamma$ with X a recurrent strictly stable process of index α and $\gamma \in (\alpha - 1, \alpha)$, generalizing the work of Engelbert and Kurenok to the asymmetric case.

Convolutions of radial, exponential densities

Sun,
Session 5

Miłosz Baraniewicz

Poitechnika Wrocławska

I will show new estimates for convolutions of multivariate, radial densities. Previous methods were limited to the situation where the second convolution of densities is comparable at infinity to the initial density. We propose a new approach which allows us to break down this barrier.

The talk is based on a paper Miłosz Baraniewicz, Kamil Kaleta: Exponential densities and compound Poisson measures, arXiv preprint 2022.

Subgeometric ergodicity of regime-switching diffusion processes

Sun,
Session 5

Petra Lazic

University of Zagreb

I will discuss subgeometric ergodicity of a class of regime-switching diffusion processes. These are processes that, beside the continuous, diffusive one, have a second, discrete component which changes the behaviour of the process at random times. The theory about them is quite interesting as it shows that in many aspects they exhibit different characteristics than classical diffusion processes. In this talk, I will derive conditions on the drift and diffusion coefficients which result in subgeometric ergodicity of the corresponding semigroup, that is, which allow us to find explicit bounds on the rate of the convergence with respect to two distance functions: the total variation distance and the class of Wasserstein distances.

Discrete Feynman–Kac operators for Markov chains with direct step property

Sun,
Session 5

Mateusz Śliwiński

Wrocław University of Science and Technology

I will present our results on decay at infinity for harmonic functions of discrete Feynman–Kac operators constructed from Markov chains. We focus on a wide class of chains with the direct step property and confining potentials. The talk will be based on the paper: Wojciech Cygan, Kamil Kaleta, Mateusz Śliwiński "Decay of harmonic functions for discrete time Feynman–Kac operators with confining potentials", available at [arXiv:2109.03788](https://arxiv.org/abs/2109.03788).

Option Pricing in Emerging Markets Using Pure Jump Processes: Explicit Calibration to BIST30 European Index Options

Sun,
Session 5

Bilgi Yilmaz

TU Kaiserslautern

In this study, we investigate the VG and NIG models' option pricing performance by comparing them with the BS option pricing model. We used the BIST30 index daily price and European Call and Put options written on this index extend from 05 May 2018 to 05 May 2020 for given exercise prices with a maturity of 90 days. In this period, the call options' strike prices range from 1200 to 1650 while the put options' strike prices range from 1000 to 1400. To compare efficiency: i) we calibrate the models by minimizing the sum of squared deviations between the observed and theoretical option prices, ii) we compute the option prices and compare the observed option prices. We observe while the NIG process performs better than both VG and BS models, the BS model is the worst in European option pricing.



Speakers

Angtuncio Hernandez, Osvaldo, 9

Bang, David, 4

Baraniewicz, Miłosz, 11

Buckland, Matthew, 9

Contreras, Jesus, 4

Gardini, Matteo, 10

Geiss, Sarah, 7

Gonzalez Cazares, Jorge, 4

Karbach, Sven, 5

Klump, Alexander, 8

Kumar, Chaman, 5

Kumar, Tejinder, 6

Lazic, Petra, 11

Panzo, Hugo, 10

Pasemann, Gregor, 6

Pedraza Ramirez, Jose Manuel, 8

Santoyo Cano, Alejandro, 11

Scheffels, David, 6

Sebek, Stjepan, 8

Szulda, Guillaume, 5

Valentic, Ivana, 8

Yilmaz, Bilgi, 12

Yuan, Shenglan, 7

Śliwiński, Mateusz, 12