# Seminar "Diskrete Finanzmathematik" (SEM 463) and Fortgeschrittenenseminar "Finanzmathematik" (MAS 540)

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The seminar covers various topics in financial modeling in discrete time, which are intended to complement the lecture course "Mathematical Finance" but other prior knowledge in mathematical finance such as the lecture course "Computational Finance" is a sufficient background for the seminar. A good knowledge in probability theory is prerequisite.

#### Requirements

- Each talk is supposed to be one hour (60 min).
- Talks can be given in English or German.
- Every talk needs to contain properly worked out mathematics.
- Slides as well as hand-written presentations are allowed.
- Prepare a hand-out of one page (A4) for the other students. A pdf-file of your hand-out needs to be send to me before your presentation.
- You need to be able to answer questions regarding your topic.
- Active participation during all talks is strongly encouraged.
- If your talk is only 40-50 minutes, you are requested to write a summary of your talk of about 5 pages. A talk of less than 40 minutes is not sufficient to pass the seminar.

#### Literature

- [FS04] Hans Föllmer and Alexander Schied: Stochastic Finance, de Gruyter, 2004.
- [HTS02] Wolfgang Härdle, Torsten Kleinow, and Gerhard Stahl: Applied Quantitative Finance, Springer-Verlag, 2002.
- [JPR07] Eric Jondeau, Ser-Huang Poon, and Michael Rockinger: *Financial modeling under non-Gaussian distributions*. Springer-Verlag, 2007.
  - [KS10] Yuri Kabanov and Mher Safarian: Markets with Transaction Costs, Springer-Verlag, 2010.
    - **[K06]** Jürgen Kremer: *Einführung in die Diskrete Finanzmathematik*, Springer-Verlag, 2006
  - [PR12] Andrea Pascucci and Wolfgang J. Runggaldier: Financial Mathematics, Springer-Verlag, 2012.
    - [R13] Ludger Rüschendorf, Mathematical Risk Analysis, Springer-Verlag, 2013.
    - [Y18] Jia-An Yan: Introduction to Stochastic Finance, Springer-Verlag, 2018.

With your university account you have access to all the above books of Springer on the webpage of Springer.

# Topics

The topics below are usually too long for your presentation. It is your task to decide, which are the key insights and results of your topic. The goal is to give a very good and understandable talk and not to present everything and very detail.

# 1 Statistical properties of financial market data

Present the essential observations of Chapter 2 of [JPR07] and explain the necessary mathematical and financial background. (The topic is for Bachelor's students only.)

# 2 Portfolio selection theory in discrete time (1)

Present Section 2.1-2.2 of [Y18]: mean-variance analysis and the capital asset pricing model (CAPM). The sections are long but mathematical not very hard. The main task is to condense the material into a one hour talk. (The topic is for Bachelor's students only.)

# 3 Portfolio selection theory in discrete time (2)

Present Section 2.3-2.7 of [Y18]: arbitrage pricing theory (APT), mean-semivariance multistage mean-variance model, model expected utility theory, consumption-based asset pricing models. The sections are long but mathematical not very hard. The main task is to condense the material into a one hour talk. (The topic is for Bachelor's students only.)

## 4 Von Neumann–Morgenstern representation of preferences

Present Section 2.1 and 2.2 of [FS04], in particular the theorem regarding the Von Neumann–Morgenstern representation of preferences. (The topic is open for Bachelor's and Master's students.)

# 5 1. FTAP in one period on general probability spaces (1)

Present Section 1.6 and the necessary results from Chapter 1 of [FS04], that is, the general first fundamental theorem of asset pricing in one period with random initial condition. This topic might be a bit long and could be re-balanced with the subsequent topic. (The topic is recommended for Master's students or Bachelor's students with an interest in functional analysis and measure theory.)

# 6 Multi-period 1. FTAP on general probability spaces (2)

Present Section 5.1 and 5.2 of [FS04], that is, the first fundamental theorem of asset pricing in a multi-period setting on general probability spaces. (The topic is recommended for Master's students or Bachelor's students with an interest in functional analysis and measure theory.)

# 7 American options in incomplete markets

Present Chapter 6 of [FS04] about the pricing and hedging of American options. The focus of the talk should be on American options in incomplete markets. (The topic is open for Bachelor's and Master's students but recommended for students who have taken the lecture course "Mathematical Finance".)

## 8 Arbitrage opportunities under hedging constraints

Present Section 9.1 of [FS04]: characterization of no arbitrage under hedging constraints. If time allows, present also the essentials of Section 9.2-9.4. (The topic is open for Bachelor's and Master's students.)

### 9 Arbitrage theory under transaction costs

Present Section 3.1 of [KS10], that is, financial models with transaction cost and the characterization of no arbitrage in these models. If time allows, the hedging problem of European (and American options) in financial markets with transaction costs could be discussed, see Section 3.3-3.4 of [KS10]. Focus should be on the case of finite probability spaces. (The topic is recommended for Master's students.)

## 10 Calibration of the CRR model and option pricing

Present Section 4.3-4.4 of [K06]. The focus should be on the calibration of the CRR model (Section 4.2). The implementation of the pricing of European options (Section 4.4) can be discussed. (The book [K06] is only available in German. The topic is for Bachelor's students only.)

## 11 Option pricing under dividend payments

Present Section 4.5 of [K06], that is, the pricing of European options under assumption that the underlying price process pays some dividend. Depending on time, discuss the case of American options as well, see Section 4.7 of [K06]. (The book [K06] is only available in German. The topic is for Bachelor's students only.)

## 12 Interest rates modelling in discrete time

Present Chapter 4 of [PR12], that is, the discrete-time models for interest rates introduced in Section 4.1-4.4. If time allows, the interest rates derivatives could be discussed, see Section 4.5. (The topic is intended for Bachelor's students.)

#### 13 Risk management and value at risk

Present the essential observations and results of Chapter 8 of [JPR07] and explain the necessary mathematical and financial background. The Chapter 8 treats various approaches to model and measure risk when trading on financial markets. (The topic is intended for Bachelor's students.)

## 14 Copulas, Sklar's theorem and value at risk

Present Section 1.1 and 1.2 of [R13], in particular, present the Sklar's theorem with its proof. Then, present the essentials of Chapter 2 in [HTS02]: how copulas can be used to calculate the value at risk. (The topic is intended for Bachelor's students.)

# 15 Optimal allocations and pareto equilibrium (1)

Present Section 10.1 of [R13], that is, the characterization of a model is in a pareto equilibrium is terms of coherent risk measures. Introduce and recall the necessary results from the previous chapters. (The topic is recommended for Master's students.)

#### 16 Optimal allocations and pareto equilibrium (2)

Present Section 10.2 of [R13], that is, the question how to allocate optimally risk when an equilibrium condition does not hold. In particular, prove Theorem 10.9. Introduce and recall the necessary results from the previous chapters. (The topic is recommended for Master's students.)

# **Research related topics**

#### 17 Neural network regression for Bermudan option pricing

Bermudan options are basically discrete-time American options. The pricing of such options is numerically rather challenging and thus there is still an on-going effort to develop novel numerical methods to price American options. The aim of the talk(s) present the paper "Neural network regression for Bermudan option pricing". It introduces a neural network based approach to the numerical pricing of American options. (The topic is recommended for a group of 2-3 students, leading to 2-3 talks.)

**Reference:** "Neural network regression for Bermudan option pricing" by Bernard Lapeyre and Jérôme Lelong;

available at https://arxiv.org/abs/1907.06474

#### 18 Volatility is rough

The volatility of a financial market is a measure how heavily stock prices fluctuate. In mathematical finance the volatility parameter is maybe the most important parameter, which has led to an extensive effort to develop advanced mathematical models describing the volatility process. The aim of the talk is to present the recent paper "Volatility is rough", which uses high frequency data to estimating smoothness of the volatility process. (This topic is a bit more advanced. Some mathematical details can be dropped.)

**Reference:** "Volatility is rough" by Jim Gatheral, Thibault Jaisson and Mathieu Rosenbaum;

available at https://www.tandfonline.com/doi/full/10.1080/14697688.2017.1393551

#### **19** Climate Impact Investing

There is an increasing effort to investigate how financial markets can help to transform the industry to become more sustainable and environment-friendly. The aim of the talk(s) is to present the recent paper "Climate Impact Investing", which investigates how green investing spurs companies to mitigate their carbon emissions by raising the cost of capital of the most carbon-intensive companies. (The topic is recommended for (Master's) students with some knowledge in stochastic calculus. While the paper is long, it can be easily divided into several parts. It is possible to take this topic as a group of 2-3 students.)

**Reference:** "Climate Impact Investing" by Tiziano De Angelis, Peter Tankov and Olivier David Zerbib;

available at https://papers.ssrn.com/sol3/papers.cfm?abstract\_id=3562534