Maß- und Integrationstheorie **Universität Mannheim**
HWS 2019 Dr. H. Pitters, Dr. Q. Shi

Week 4

We say $A \subset \mathbb{R}^2$ is (Lebesgue)-measurable in the sense of *Definition 2.29*.

Exercise 1. Calculate the exterior measure of the cantor set $\lambda^*(\mathcal{C})$.

Exercise 2. Let $A \subset \mathbb{R}^2$.

(i) Prove the identity:

$$\lambda^*(A) = \inf \left\{ \lambda^*(O) \colon A \subset O \subset \mathbb{R}^2 \text{ and } O \text{ is open.} \right\}$$

(ii) A set $B \subset \mathbb{R}^2$ is called a G_δ-set, if it is a countable intersection of open sets, i.e. we can write $B = \bigcap_{n=1}^\infty O_n$ for some sequence of open sets (O_n). Show that, there exists a G_δ-set $B \supset A$, such that $\lambda^*(B) = \lambda^*(A)$.

Exercise 3. Let $A \subset \mathbb{R}^2$. Prove that, the following three statements are equivalent:

(i) A is measurable.

(ii) For any $\epsilon > 0$, there exists an open set $O \supset A$ such that $\lambda^*(O \setminus A) < \epsilon$.

(iii) For every $B \subset \mathbb{R}^2$, there is the identity $\lambda^*(B \cap A) + \lambda^*(B \cap A^c) = \lambda^*(B)$.

Exercise 4. Show that, if $\lambda^*(A) = 0$, then A is measurable.

Exercise 5. Who are they?