

Introduction to PDEs

Homework Assignment 2

1. Read lecture notes chapter 1 and answer problem set 1 on page 8.
2. Use the method of characteristics to solve the following problem

$$\begin{cases} u_t + (1 + x^2)u_x - u = 0 & x \in \mathbb{R} \\ u(x, 0) = \arctan(x) \end{cases}$$

3. Let $u(x)$ be a continuous function in \mathbb{R}^n . For all $x \in \mathbb{R}^n, r > 0$ let $B(x, r) = \{y \in \mathbb{R}^n : |y - x| < r\}$ and $\partial B(x, r) = \{y \in \mathbb{R}^n : |y - x| = r\}$ denote the n -Dimensional ball with radius r and its boundary. Prove that

$$u(x) = \lim_{r \rightarrow 0^+} \int_{\partial B(x, r)} u(y) dS_y = \lim_{r \rightarrow 0^+} \int_{B(x, r)} u(y) dy$$

4. Reproduce the Kirchhoff formula.

Due September 18, before 12:00 in Box 46216