

## Introduction to PDEs

1. Find the classical solution of the following problems and point out the lifespan of the solution respectively.

$$(a) \begin{cases} u_t + uu_x = 0, & x \in \mathbb{R}, t > 0, \\ u|_{t=0} = x. \end{cases}$$

$$(b) \begin{cases} u_t - uu_x = 0, & x \in \mathbb{R}, t > 0, \\ u|_{t=0} = x. \end{cases}$$

2. (Traffic flow in tunnel found by Greenberg in 1959) Study the traffic flow problem with flux

$$q(\rho) = v_m \rho \ln \frac{\rho_m}{\rho},$$

and initial data

$$\rho(x, 0) = \begin{cases} \rho_l, & x < 0, \\ \rho_r, & x > 0, \end{cases}$$

where  $v_m$  is the maximum velocity,  $\rho_m$  is the maximum density and  $\rho_l = \frac{1}{2}\rho_m$ ,  $\rho_r = \frac{1}{3}\rho_m$ . Give the solution, and draw a picture for partial path of car trajectories.

**Due to 04.12 20:00 in box 46216**