Lehrstuhl für Mathematik IV
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13.11.2019

Introduction to PDEs

1. Let Ω be an open subset of $\mathbb{R}^n$. The Neumann boundary value problem of Poisson's equation is given by
$$\begin{cases} -\Delta u = f, & \text{in } \Omega, \\ \nabla u \cdot \gamma\big|_{\partial\Omega} = h. \end{cases}$$
Try to give the definition of the Green's function of Neumann boundary value problem. Furthermore give the solution formula by using the Green's function you have defined.

2. The Poisson's formula for the half space problem is given by
$$u(x) = \frac{2x_n}{n\alpha(n)} \int_{\partial\mathbb{R}^n_+} \frac{g(y)}{|x-y|^n}\, dy.$$
Prove that
$$-\Delta u(x) = 0, \quad \forall\, x \in \mathbb{R}^n_+.$$

3. The Poisson kernel for the half space problem is given by
$$K(x,y) := \frac{2x_n}{n\alpha(n)|x-y|^n}.$$
Prove for $n = 2$ that
$$\int_{\partial\mathbb{R}^2_+} K(x,y)dy = 1.$$

4. The Poisson's formula for the unit ball problem is given by
$$u(x) = \frac{1-|x|^2}{n\alpha(n)} \int_{\partial B(0,1)} \frac{h(y)}{|x-y|^n}\, dS_y.$$
Prove in two dimensions ($n = 2$) that for $h \in C(\partial B(0,1))$, it holds
$$\lim_{x\to x_0} u(x) = h(x_0), \quad \forall\, x_0 \in \partial B(0,1).$$

Due to 20.11 20:00 in box 46216