

Introduction to PDEs

1. Prove that

$$\Phi(x, y) = -\frac{1}{2\pi} \ln |x - y|$$

is the fundamental solution of Poisson's equation in two dimension, i.e., $\forall y \in \mathbb{R}^2$, prove that $\Phi(x, y)$ is a solution of $-\Delta_x u = \delta(x - y)$ in the sense of distribution.

2. Modify the proof of the mean value formula to show for $n \geq 3$ that

$$u(0) = \int_{\partial B(0,r)} g(x) dS_x + \frac{r^2}{2n},$$

provided that $\begin{cases} -\Delta u = 1, & x \in B(0, r) \\ u = g, & x \in \partial B(0, r) \end{cases}$.

Important announcement: Lecture on 31.10, B2 will be replaced by tutorial in lecture room A203. Tutorial on 31.10, B1 will be cancelled. The missing two lectures will be done in November.

Due to 13.11 20:00 in box 46216