Lehrstuhl für Mathematik IV
Prof. boshi Li Chen

Universität Mannheim
Fakultät für Wirtschaftsinformatik
und Wirtschaftsmathematik

Tutor: Timo Neef

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Introduction to PDEs

1. Solve the following initial boundary value problem

$$\begin{aligned} u_t - u_{xx} &= x(1-x), \qquad && (x,t) \in (0,1) \times (0,\infty), \\ u|_{t=0} &= \sin(\pi x), && x \in [0,1], \\ u|_{x=0} = 0, \quad u_x|_{x=1} &= -\pi, && t > 0. \end{aligned}$$

   Hint: use $v = u + \pi x$ to homogenize the boundary condition

2. Let $Q_T = (0,l) \times (0,T]$. Suppose that $u(x,t) \in C^{2,1}(Q_T) \cap C(\overline{Q}_T)$ is the solution to

$$\begin{aligned} u_t - u_{xx} &= f(x,t), \quad && (x,t) \in (0,l) \times (0,T], \\ u|_{t=0} &= \varphi(x), && x \in [0,l], \\ u|_{x=0} = u|_{x=l} &= 0, && t \in (0,T]. \end{aligned}$$

   Prove that for $0 < t \leq T$, there holds

$$\int_0^l u_x^2(x,t)dx + \int_0^t \int_0^l u_t^2(x,s)dxds \leq \int_0^l \varphi_x^2(x)dx + \int_0^t \int_0^l f^2(x,s)dxds.$$

3. Let $Q_T = (0,1) \times (0,T]$. Suppose that $u(x,t) \in C^{2,1}(Q_T) \cap C(\overline{Q}_T)$ satisfies

$$\mathcal{L}u = u_t - a^2 u_{xx} + c(x,t)u \leq 0, \quad (x,t) \in Q_T,$$

   where $c(x,t) \geq 0$. Prove that

$$\max_{\overline{Q}_T} u(x,t) \leq \max_{\partial_p Q_T} u^+(x,t),$$

   where $u^+(x,t) = \max\{u(x,t), 0\}$.
   Hint: consider the case $\mathcal{L}u < 0$ first, then construct an auxiliary function to complete the proof

Due to 30.10 20:00 in box 46216