

Introduction to PDEs

1. Let

$$Q_n = \begin{cases} n^2x + n, & -\frac{1}{n} \leq x \leq 0, \\ -n^2x + n, & 0 < x \leq \frac{1}{n}, \\ 0, & \text{otherwise.} \end{cases}$$

Prove that

$$\int_{\mathbb{R}} Q_n(x)\phi(x) dx \rightarrow \langle \delta_0, \phi \rangle, \quad \forall \phi \in \mathcal{D}.$$

2. Verify the following equalities in the sense of distribution

(a) $\rho(x)\delta'(x) = -\rho'(0)\delta(x) + \rho(0)\delta'(x)$,

(b) $x\delta^{(m)}(x) = -m\delta^{(m-1)}(x)$,

(c) * $x^m\delta^{(m)}(x) = (-1)^m m!\delta(x)$,

(d) $(H(x)\rho(x))' = \delta(x)\rho(0) + H(x)\rho'(x)$,

where $H(x)$ is the Heaviside function and $\rho \in C^\infty(\mathbb{R})$.

3. * Calculate the following weak derivatives

(a) $(H(x)e^x)''$

(b) $f(x)'$, where $f(x) = \begin{cases} x^2, & |x| \leq 1, \\ 0, & |x| > 1. \end{cases}$

4. * Find the Fourier transform of Delta function $\delta_a(x) = \delta(x - a)$, where $a \in \mathbb{R}$.

Due to 16.10 20:00 in box 46216