Lehrstuhl für Mathematik IV
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Introduction to PDEs

1. Check the properties of Fourier transform
   (a) $(\partial_{x_j} f)^\wedge(k) = ik_j \hat{f}(k)$,
   (b) $(x_j f)^\wedge(k) = i\partial_{k_j} \hat{f}(k)$,
   (c) $f(x-a)^\wedge(k) = e^{-ia\cdot k} \hat{f}(k)$,
   (d) $(f(\lambda x))^\wedge(k) = \frac{1}{|\lambda|^n} \hat{f}(\frac{k}{\lambda}), \forall \lambda \neq 0$,
   (e) $(f * g)^\wedge(k) = (2\pi)^{\frac{n}{2}} \hat{f}(k)\hat{g}(k)$.
2. Compute the Fourier transform of $f(x) = e^{-|x|}$, $x \in \mathbb{R}$ and hence use the inverse Fourier transform to compute $\int_0^\infty \frac{1}{1+k^2}\, dk$ and $\int_0^\infty \frac{k \sin kx}{1+k^2}\, dk, x > 0$.
3. (a) Compute the Fourier transform of $f(x) = e^{-x^2}$, $x \in \mathbb{R}$.
   (b) Compute the inverse Fourier transform of $f(k) = e^{-k^2 t}$, $k \in \mathbb{R}$.
   Hint: use the scaling property $(f(\lambda x))^\wedge = \frac{1}{|\lambda|} \hat{f}(\frac{k}{\lambda})$

Due to 09.10 20:00 in box 46216