Lehrstuhl für Mathematik IV
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Introduction to PDEs

1. Solve the eigenvalue problem

$$\begin{aligned} X''(x) + \lambda X(x) = 0, &\quad x \in (0, l), \\ X(0) = X'(l) = 0. & \end{aligned}$$

2. Now we consider the following initial boundary value problem

$$\begin{aligned} u_{tt} - u_{xx} = 0, &\qquad (x,t) \in (0,1) \times (0,\infty), \\ u|_{x=0} = u|_{x=1} = 0, &\qquad t \geq 0, \\ u|_{t=0} = \alpha x^4 + \beta x^3 + \sin \pi x, \quad u_t|_{t=0} = \gamma \cos \pi x, &\qquad x \in [0,1]. \end{aligned}$$

   (a) Apply separation of variables to derive the solution formula,
   (b) Determine α, β and γ so that the solution you have derived is indeed a classical solution.

3. Use energy estimate to prove that the following initial boundary value problem

$$\begin{cases} u_{tt} - a^2 u_{xx} = 0, & (x,t) \in (0,l) \times \mathbb{R}_+, \\ u|_{t=0} = 0, \quad u_t|_{t=0} = 0, & \\ \alpha_1 u|_{x=0} - \beta u_x|_{x=0} = 0, & \alpha_1, \alpha_2, \beta > 0, \\ \alpha_2 u|_{x=l} + \beta u_x|_{x=l} = 0 & \end{cases}$$

   has only trivial solution $u \equiv 0$.

Due to 02.10 20:00 in box 46216