Lehrstuhl für Mathematik IV
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Introduction to PDEs

1. Suppose that u is the solution of

$$\begin{cases} u_{tt} - \Delta u = 0, \quad (x,t) \in \mathbb{R}^n \times (0, +\infty), \\ u|_{t=0} = g(x), \\ u_t|_{t=0} = h(x). \end{cases}$$

Show that the energy estimates hold in multi-dimensional case ($n \geq 2$).

2. (Equipartition of energy) Let $u \in C^2(\mathbb{R} \times [0, \infty))$ solve the initial value problem

$$\begin{aligned} u_{tt} - u_{xx} &= 0, \qquad (x,t) \in \mathbb{R} \times (0, \infty), \\ u|_{t=0} = g, \quad u_t|_{t=0} &= h, \qquad x \in \mathbb{R}, \end{aligned}$$

where g and h have compact support. We define the kinetic energy as

$$k(t) = \frac{1}{2} \int_{-\infty}^{\infty} u_t^2(x,t)dx$$

and the potential energy as

$$p(t) = \frac{1}{2} \int_{-\infty}^{\infty} u_x^2(x,t)dx.$$

Prove that

(a) $k(t) + p(t)$ is a constant in t.

(b) $k(t) = p(t)$ for large enough time t.

3. Apply separation of variables to get formal solution of

$$\begin{aligned} u_{tt} &= a^2 u_{xx}, & &(x,t) \in (0,l) \times (0,\infty),\ a > 0, \\ u|_{x=0} &= u|_{x=l} = 0, & &t \geq 0, \\ u|_{t=0} = \sin \tfrac{3\pi x}{l}, \quad u_t|_{t=0} &= x(l-x), & &x \in [0,l]. \end{aligned}$$

Due to 25.09 12:00 in box 46216