$\frac{1}{4} < xy < 4 \qquad \frac{1}{3} < \frac{y}{x} < 3$

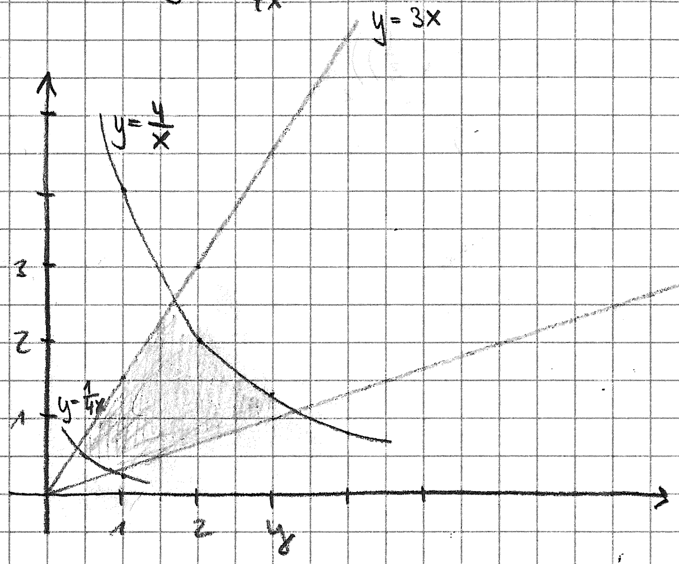
$y < \frac{4}{x} \qquad y < 3x$ ✓

$x > \frac{y}{3}$

$x < \frac{4}{y} \qquad y > \frac{x}{3}$ ✓

$x > \frac{1}{4y}$

$y > \frac{1}{4x}$




$u = xy \qquad v = \frac{y}{x}$

$\Leftrightarrow x = \frac{u}{y}$ (in v)

$v = \frac{y^2}{u}$

$y^2 = v \cdot u$

$y = \sqrt{v \cdot u}$ (in x)

$x = \frac{u}{\sqrt{v u}} = \sqrt{\frac{u^2}{v \cancel{u}}} = \sqrt{\frac{u}{v}}$

$\Phi : \mathbb{R}^+ \times \mathbb{R}^+ \longrightarrow \mathbb{R}^2, \quad (u,v) \longmapsto (x(u,v), y(u,v)) = \left(\sqrt{\frac{u}{v}}, \sqrt{uv}\right)$

$$\Phi'(u,v) = \begin{pmatrix} \frac{1}{2} \frac{1}{\sqrt{uv}} & -\frac{1}{2} \frac{\sqrt{u}}{v^{\frac{3}{2}}} \\ \frac{1}{2} \sqrt{\frac{v}{u}} & \frac{1}{2} \sqrt{\frac{u}{v}} \end{pmatrix}$$

$$\det(\Phi') = \frac{1}{4} \frac{1}{\sqrt{\cancel{u}v}} \sqrt{\frac{\cancel{u}}{v}} + \frac{1}{4} \sqrt{\frac{v}{\cancel{u}}} \frac{\cancel{\sqrt{u}}}{v^{\frac{3}{2}}}$$

$$= \frac{1}{4v} + \frac{1}{4v} = \frac{1}{2v} \neq 0$$

Mit Jacobitransformation ergibt sich:

$$\mu(M) = \int_M 1 \, d\mu = \int_R 1 \circ \Phi \cdot |\det \Phi'| \, d\mu$$

$$= \int_R \frac{1}{2v} \, d\mu = \frac{1}{2} \int_{u=1/4}^{4} \int_{v=\frac{1}{3}}^{3} \frac{1}{v} \, dv du$$

$$= \frac{1}{2} \left(4 - \frac{1}{4}\right) \left[ \ln(v) \right]_{v=\frac{1}{3}}^{3}$$

$$= \frac{15}{8} \cdot \underbrace{\left( \ln(3) - \ln\left(\frac{1}{3}\right) \right)}_{2\ln(3)}$$

$$= \frac{15}{4} \ln(3)$$